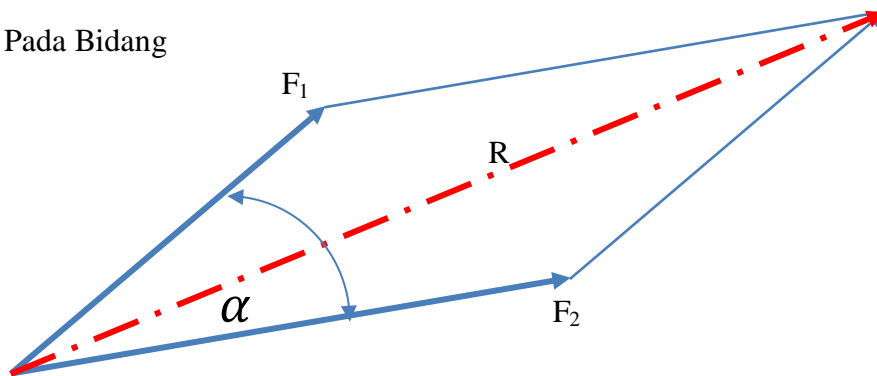


GAYA

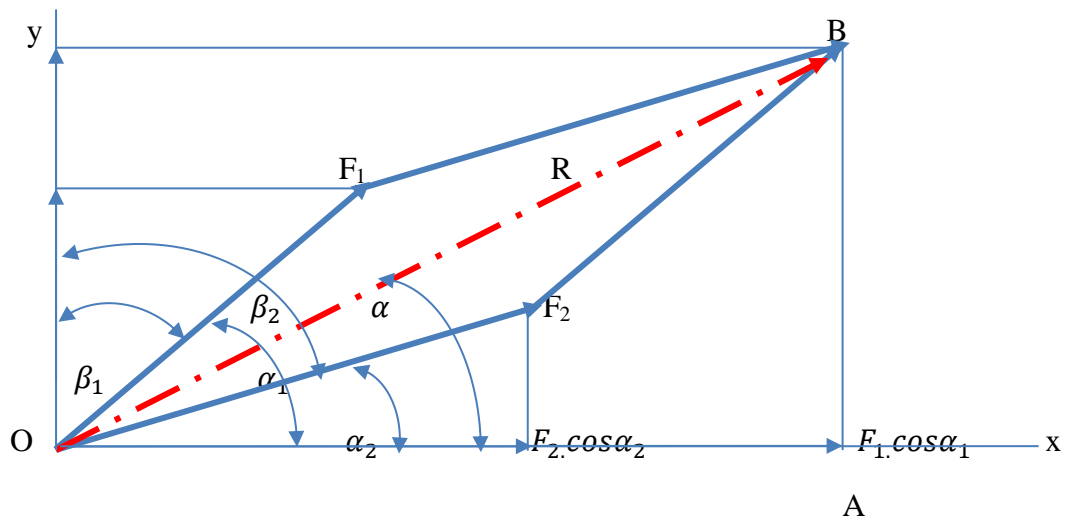
A. Gaya Pada Bidang



Resultante (R):

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cdot \cos\alpha}$$

Jika diketahui lebih dari dua gaya, maka yang digunakan adalah rumus sebagai berikut:



$$\sum F_x = R \cos \alpha = F_1 \cos \alpha_1 + F_2 \cos \alpha_2$$

$$\sum F_y = R \cos \beta = F_1 \cos \beta_1 + F_2 \cos \beta_2$$

Dalam ΔOAB :

$$R^2 = OA^2 + AB^2$$

$$R^2 = \sum F_x^2 + \sum F_y^2$$

$$R = \sqrt{\sum F_x^2 + \sum F_y^2}$$

$$\text{tg } \alpha = \frac{\sum F_y}{\sum F_x}$$

Contoh:

Diketahui:

$$\alpha_1 = 45^0 \quad \beta_1 = 45^0$$

$$\alpha_2 = 15^0 \quad \beta_2 = 75^0$$

$$F_1 = 100 \text{ N} \quad F_2 = 200 \text{ N}$$

Ditanyakan:

Besarnya resultante dan sudut yang dibentuknya terhadap sumbu x

Jawab:

$$\sum F_x = F_1 \cos \alpha_1 + F_2 \cos \alpha_2$$

$$= 100 \cdot \cos 45^0 + 200 \cdot \cos 15^0$$

$$= 100 \cdot 0,7071 + 200 \cdot 0,9659$$

$$= 70,71 + 193,3$$

$$= \mathbf{264,01 \text{ N}}$$

$$\sum F_y = F_1 \cos \beta_1 + F_2 \cos \beta_2$$

$$= 100 \cdot \cos 45^0 + 200 \cdot \cos 75^0$$

$$= 100 \cdot 0,7071 + 200 \cdot 0,2588$$

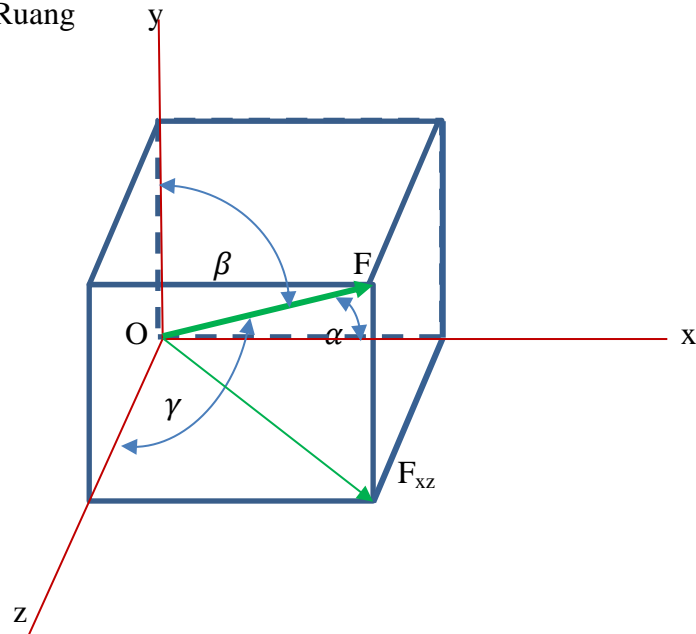
$$= 70,71 + 51,76$$

$$= \mathbf{122,47 \text{ N}}$$

$$\mathbf{R} = \sqrt{\sum F_x^2 + \sum F_y^2} = \sqrt{264,01^2 + 122,47^2} = \sqrt{\quad}$$

$$\mathbf{\text{tg } \alpha = \frac{F_y}{F_x} = \frac{122,47}{264,01} = 0,4626 \quad \text{Jadi } \alpha = 24^0 50^1}$$

B. Gaya Pada Ruang



α : sudut F terhadap sumbu x

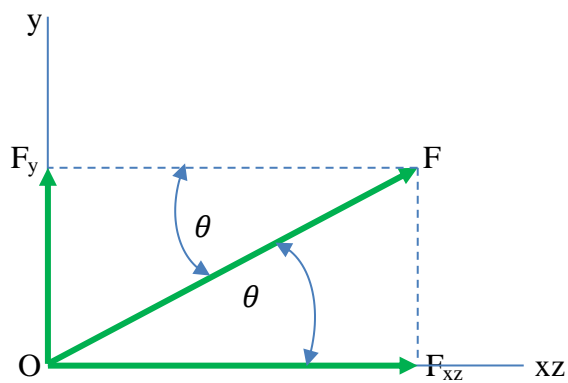
β : sudut F terhadap sumbu y

γ : sudut F terhadap sumbu z

Maka : $F_x = F \cos \alpha$

$$F_y = F \cos \beta$$

$$F_z = F \cos \gamma$$

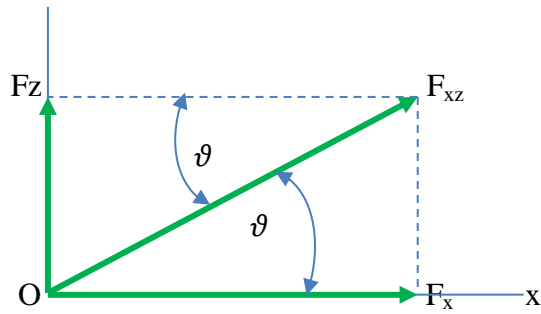


θ : sudut F terhadap sumbu xz

$$F_{xz} : F \cos \theta$$

$$F_y : F \sin \theta$$

$$F^2 = F_{xz}^2 + F_y^2 \dots\dots\dots (1)$$



ϑ : sudut F_{xz} terhadap sumbu z

$$F_x : F_{xz} \cos \vartheta$$

$$F_z : F_{xz} \sin \vartheta$$

$$F_{xz}^2 = F_z^2 + F_x^2 \dots\dots\dots(2)$$

Dari 1 da 2 didapat:

$$F^2 = F_x^2 + F_y^2 + F_z^2$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

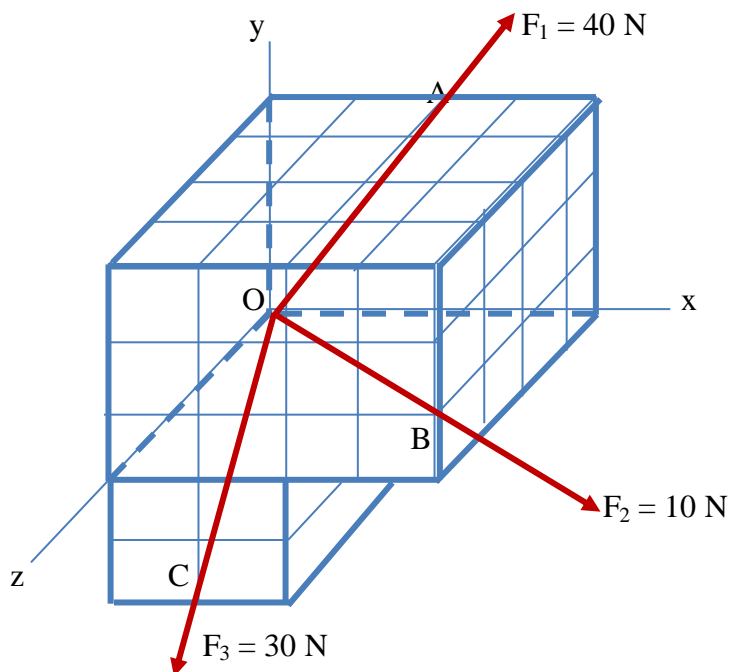
Jadi besarnya Resultante (R):

$$R = \sqrt{\sum F_x^2 + \sum F_y^2 + \sum F_z^2}$$

$$\cos \alpha = \frac{\sum F_x}{R} \quad \cos \beta = \frac{\sum F_y}{R} \quad \cos \gamma = \frac{\sum F_z}{R}$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

Contoh:



$$OA = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{2^2 + 3^2 + 0^2}$$

$$= 3,6$$

$$OB = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{4^2 + 1^2 + 4^2}$$

$$= 5,7$$

$$OC = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{1^2 + (-2)^2 + 4^2}$$

$$= 4,6$$

$$\sum F_x = F_1 \cos \alpha_1 + F_2 \cos \alpha_2 + F_3 \cos \alpha_3$$

$$= 40 * \frac{2}{3,6} + 10 * \frac{4}{5,7} + 30 * \frac{1}{4,6}$$

$$= 22 + 7 + 6,54 = \mathbf{35,54 \text{ N}}$$

$$\sum F_y = F_1 \cos \beta_1 + F_2 \cos \beta_2 - F_3 \cos \beta_3$$

$$= 40 * \frac{3}{3,6} + 10 * \frac{1}{5,7} + 30 * \frac{-2}{4,6}$$

$$= 33,2 + 1,75 - 13,1 = \mathbf{21,85 \text{ N}}$$

$$\sum F_z = F_1 \cos \gamma_1 + F_2 \cos \gamma_2 + F_3 \cos \gamma_3$$

$$= 40 * \frac{0}{3,6} + 10 * \frac{4}{5,7} + 30 * \frac{4}{4,6}$$

$$= 0 + 7 + 26,1 = \mathbf{33,1 \text{ N}}$$

$$R = \sqrt{\sum F_x^2 + \sum F_y^2 + \sum F_z^2}$$

$$= \sqrt{35,54^2 + 21,85^2 + 33,1^2}$$

$$= 53,3 \text{ N}$$

Sudut yang dibentuk oleh R:

$$\cos \alpha = \frac{\sum F_x}{R} = \frac{35,54}{53,3} = 0,6668, \text{ maka } \alpha = 48^0 11^1$$

$$\cos \beta = \frac{\sum F_y}{R} = \frac{21,85}{53,3} = 0,4099, \text{ maka } \beta = 65^0 48^1$$

$$\cos \gamma = \frac{\sum F_z}{R} = \frac{33,1}{53,3} = 0,6210, \text{ maka } \gamma = 51^0 36^1$$

Chek:

$$\underline{\underline{\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1}}$$

$$0,6668^2 + 0,4099^2 + 0,6210^2 = 1$$

$$0,9982 = 1$$

KESETIMBANGAN & DIAGRAM BENDA BEBAS

Dalam memecahkan persoalan kesetimbangan benda tegar, sangatlah penting untuk meninjau semua gaya yang beraksi pada benda tersebut, juga sama pentingnya untuk tidak memasukan setiap gaya yang tidak langsung diterapkan pada benda itu. Jadi langkah pertama dalam memecahkan persoalan adalah *penggambaran diagram benda bebas* dari benda tegar yang sedang ditinjau.

Reaksi dapat dikelompokkan menjadi tiga menurut jenis dukungan atau sambungan.

1. Reaksi yang ekuivalen dengan sebuah gaya yang diketahui garis aksinya
2. Reaksi yang ekuivalen dengan gaya yang arahnya tidak diketahui
3. Reaksi yang ekuivalen dengan suatu gaya dan suatu kopel.

Syarat Setimbang:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

Jika dalam ruang ditambah:

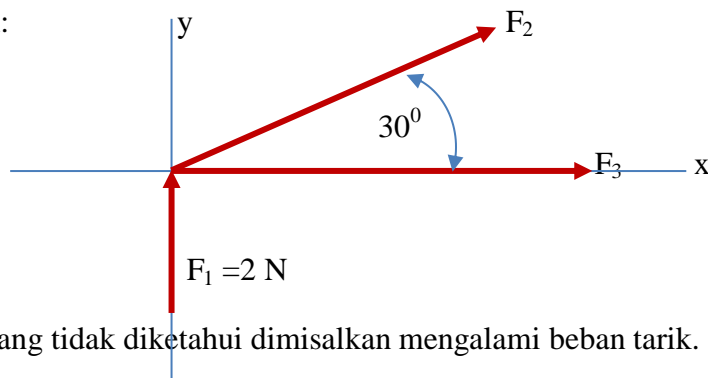
$$\sum F_z = 0$$

Jika ada unsur momen, maka:

$$\sum M = 0$$

Contoh:

(1)



Gaya yang tidak diketahui dimisalkan mengalami beban tarik.

$$\sum F_y = 0$$

$$F_1 + F_2 \sin 30^\circ = 0$$

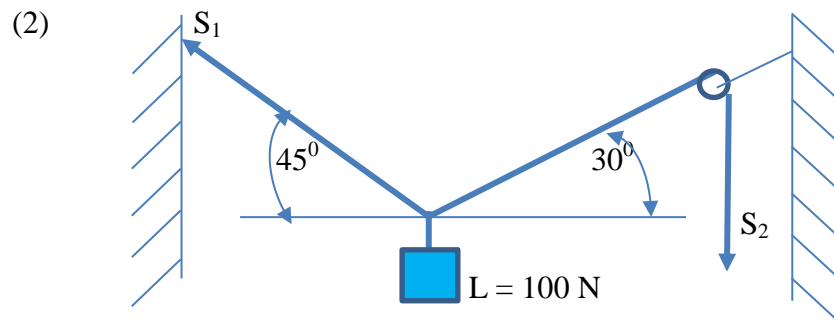
$$2 + F_2 \cdot 0,5 = 0$$

$$2 = - 0,5 \cdot F_2$$

$$F_2 = - 4 \text{ N (beban tekan)}$$

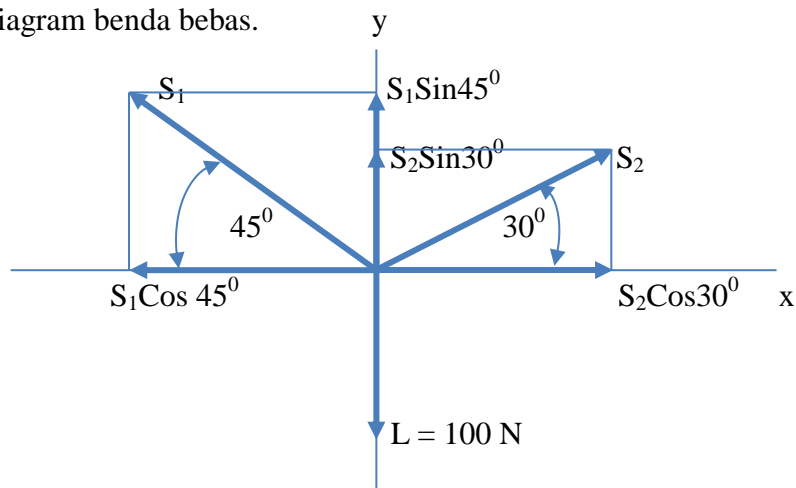
$$\sum F_x = 0$$

- $F_2 \cos 30^\circ + F_3 = 0$
- $4 \cdot 0,5\sqrt{3} + F_3 = 0$
- $F_3 = 2\sqrt{3}$ (beban tarik)**



Jawab.

Diagram benda bebas.



$$\sum F_x = 0$$

$$-S_1 \cos 45^\circ + S_2 \cos 30^\circ = 0 \quad \dots\dots\dots(1)$$

$$\sum F_y = 0$$

$$S_1 \sin 45^\circ + S_2 \sin 30^\circ - 100 = 0 \quad \dots\dots\dots(2)$$

Dari (1) dan (2) :

$$-S_1 \cdot 0,7071 + S_2 \cdot 0,8660 = 0$$

$$\underline{S_1 \cdot 0,7071 + S_2 \cdot 0,5 - 100 = 0}$$

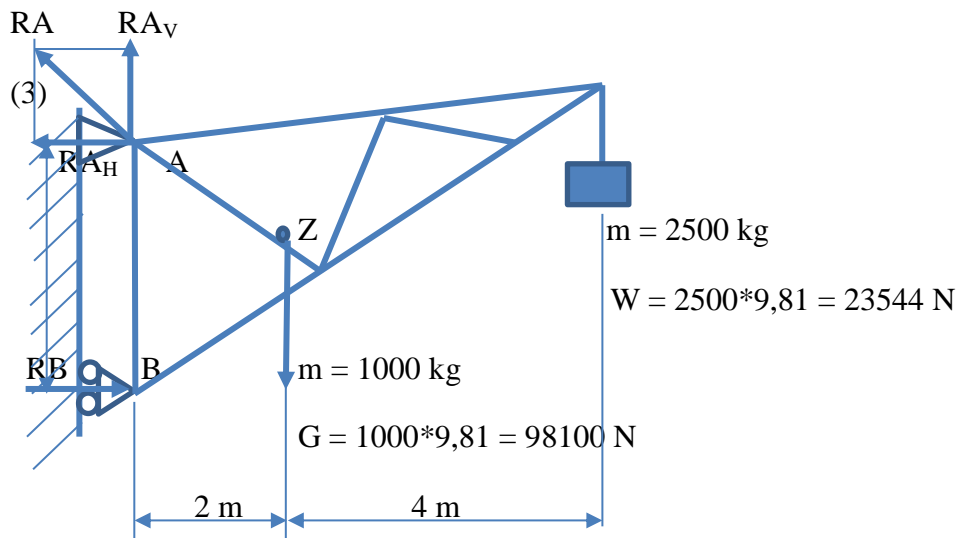
$$S_2 \cdot 1,3660 = 100$$

$$\mathbf{S_2 = 73,2 \text{ N}}$$

$$S_1 \cdot 0,7071 + 73,2 \cdot 0,5 - 100 = 0$$

$$\mathbf{S_1 = 89,66 \text{ N}}$$

(3)



$$\sum M_A = 0$$

$$-R_B \cdot 1,5 + G \cdot 2 + W \cdot 6 = 0$$

$$R_B \cdot 1,5 - 9810 \cdot 2 - 23544 \cdot 6 = 0$$

$$R_B \cdot 1,5 - 19620 - 141264 = 0$$

$$R_B = \frac{160884}{1,5} = 1077256 \text{ N}$$

$$\sum M_B = 0$$

$$- R_{Ax} \cdot 1,5 + G \cdot 2 + W \cdot 6 = 0$$

$$- R_{Ax} \cdot 1,5 + 98100 \cdot 2 + 23544 \cdot 6 = 0$$

$$R_{Ax} = -\frac{160884}{1,5} = 1077256 \text{ N}$$

$$\sum F_y = 0$$

$$R_{Ay} + G + W = 0$$

$$R_{Ay} + 9810 + 23544 = 0$$

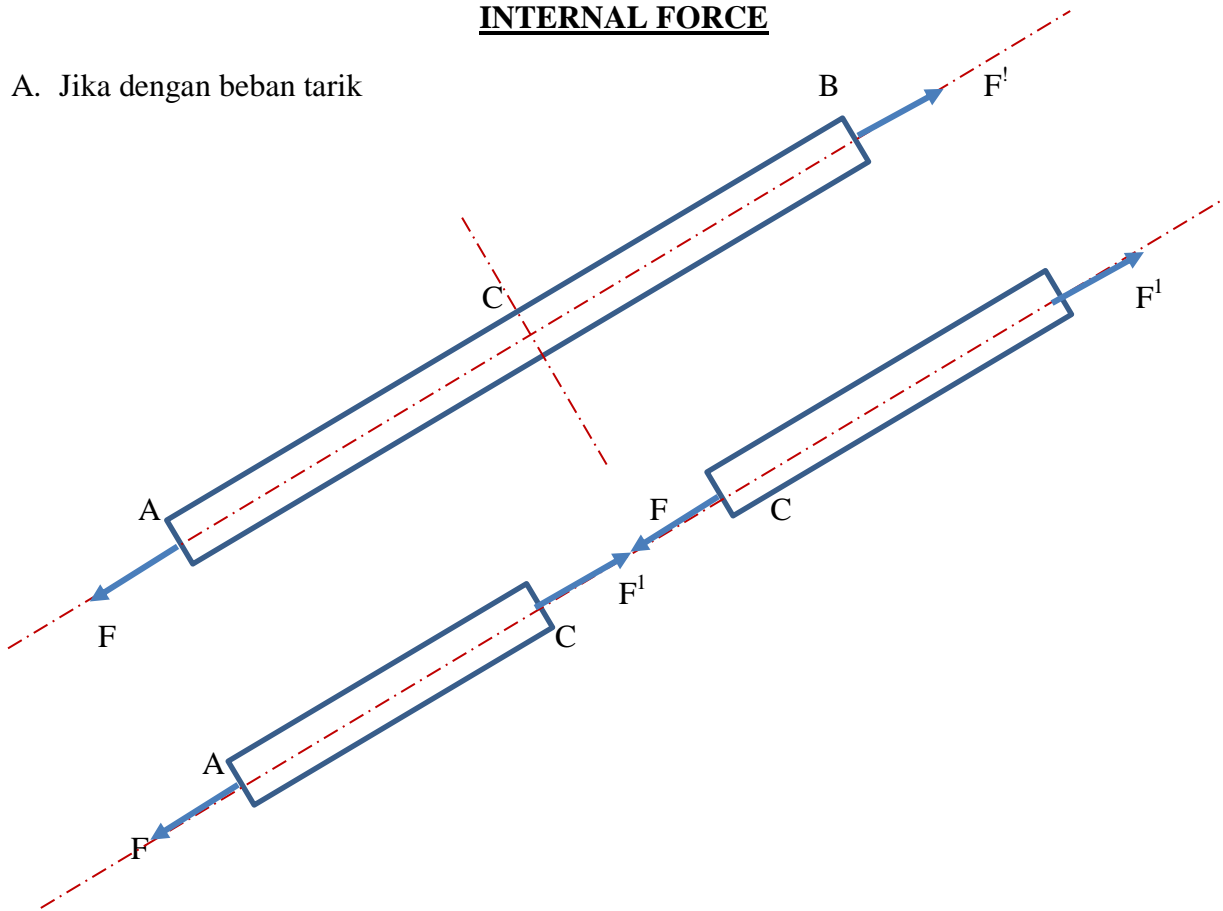
$$R_{Ay} = 33354 \text{ N}$$

$$\text{tg } \alpha = \frac{R_{Ay}}{R_{Ax}} = \frac{33354}{107256} = 0,3110$$

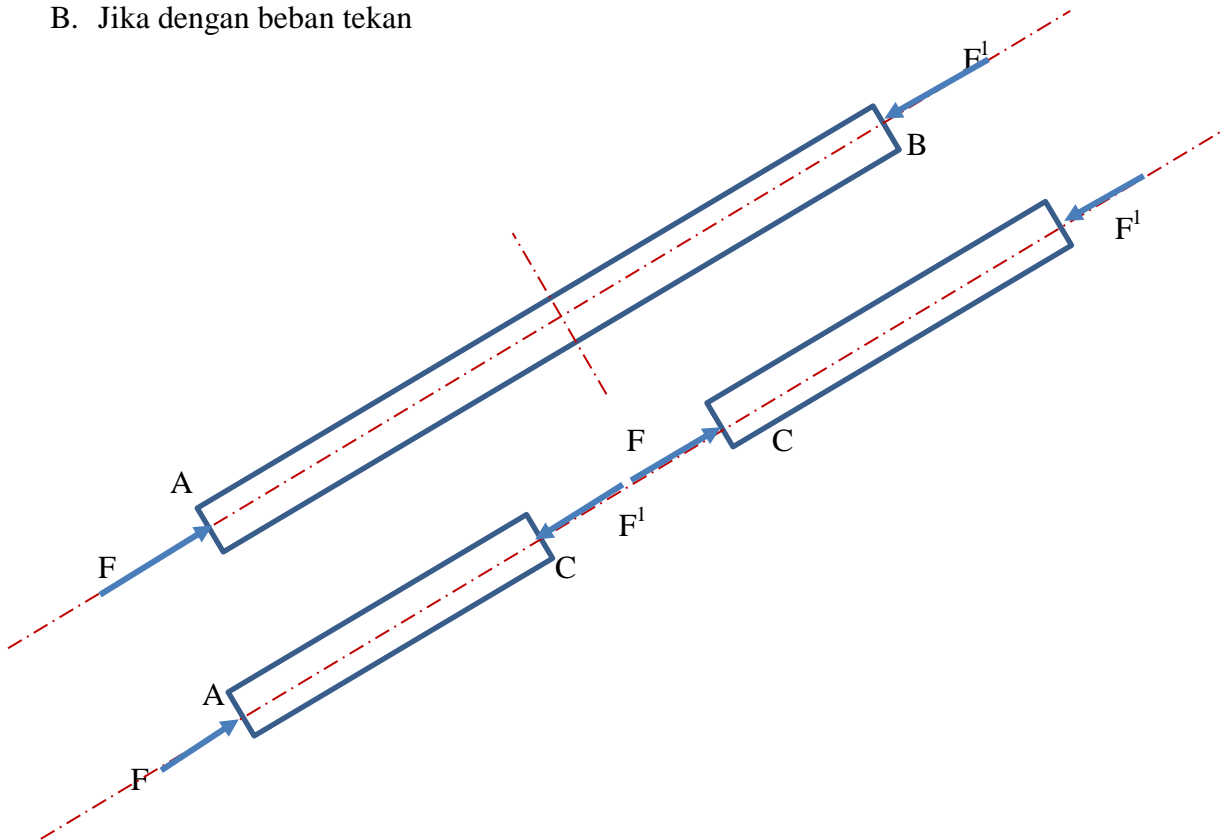
$$\alpha = 17^0 17^1$$

INTERNAL FORCE

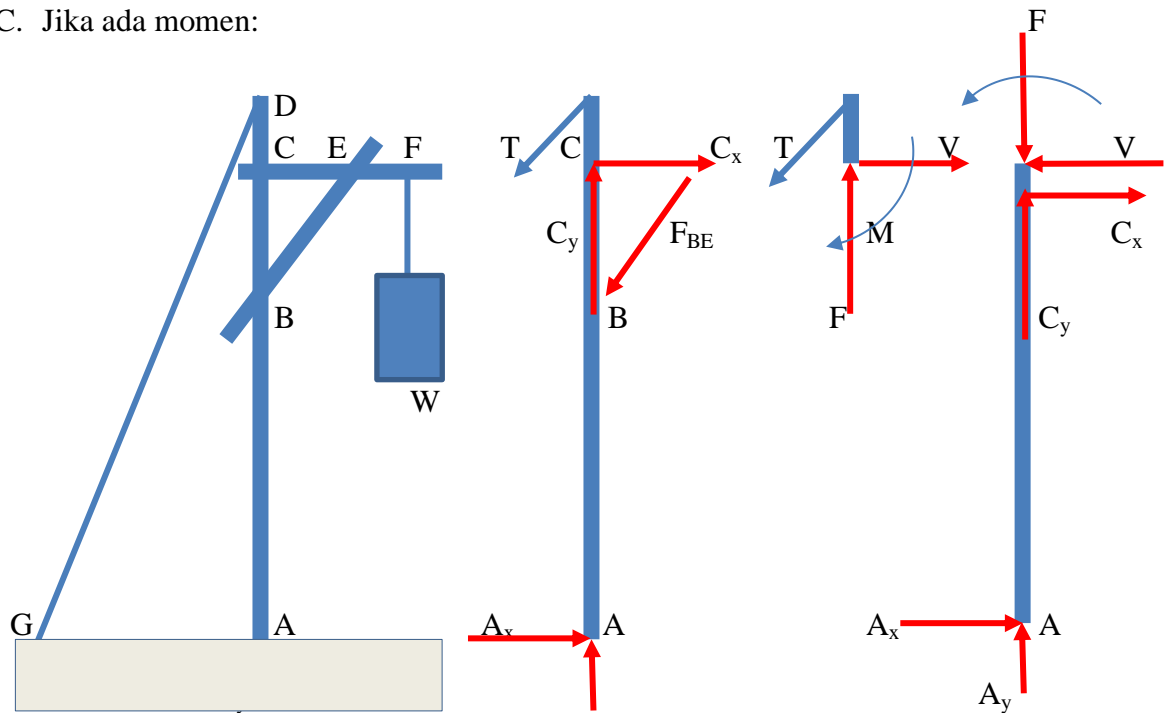
A. Jika dengan beban tarik



B. Jika dengan beban tekan



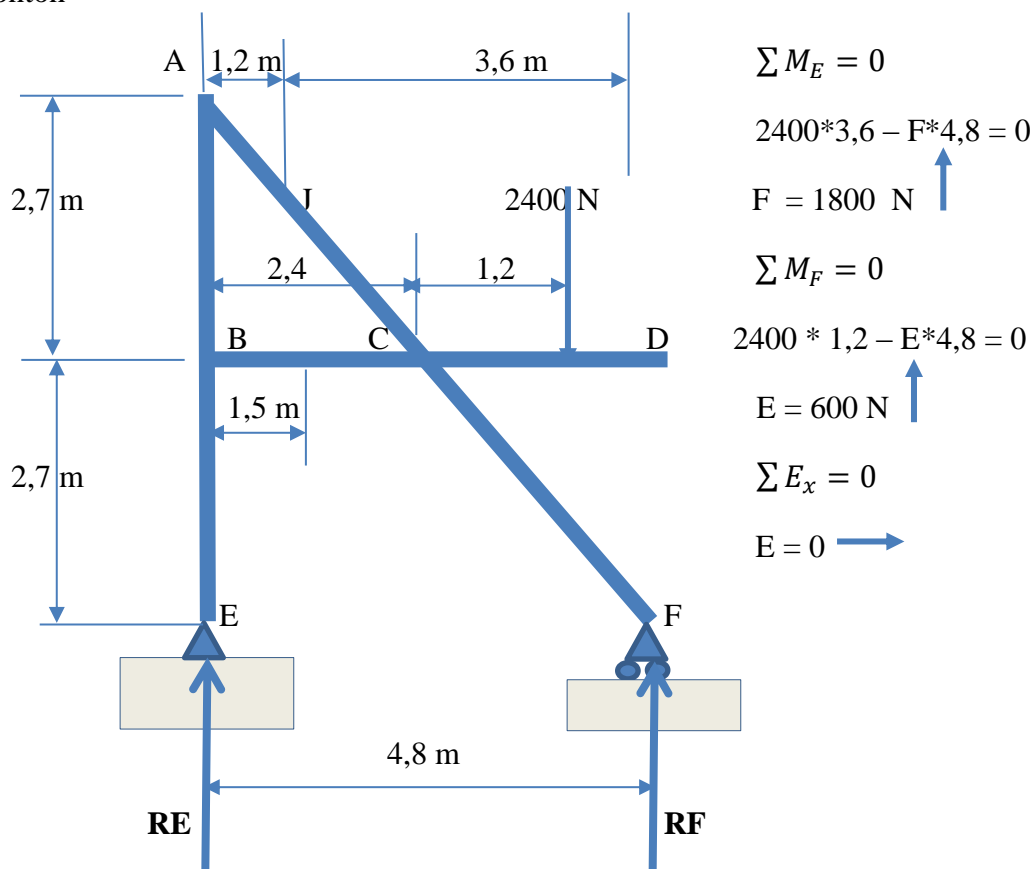
C. Jika ada momen:

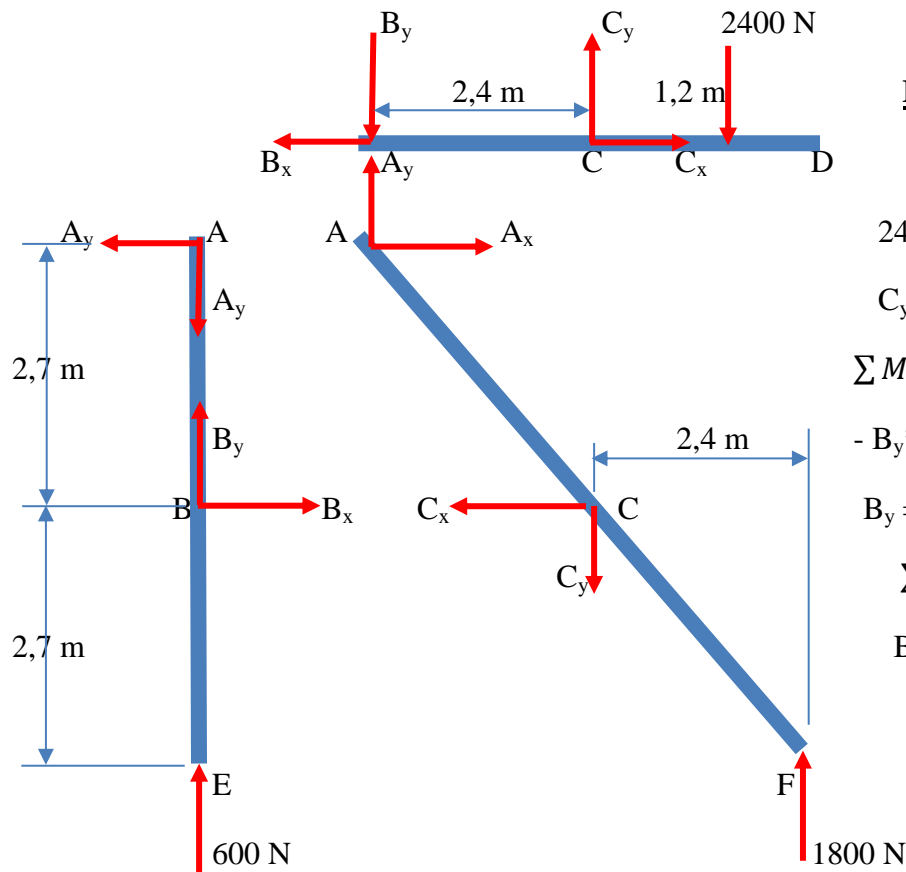


F: Gaya aksial

C: Gaya geser

Contoh





Bagian BCD:

$$\sum M_B = 0$$

$$2400 \cdot 3,6 - C_y \cdot 2,4 = 0$$

$$C_y = 3600 \text{ N} \quad \uparrow$$

$$\sum M_C = 0$$

$$-B_y \cdot 2,4 + 2400 \cdot 1,2 = 0$$

$$B_y = 1200 \text{ N} \quad \uparrow$$

$$\sum F_x = 0$$

$$B_x - C_x = 0$$

Bagian ABE:

$$\sum M_A = 0$$

$$B_x \cdot 2,7 = 0 \quad \longrightarrow \quad B_x = 0$$

$$\sum F_x = 0$$

$$A_x + B_x = 0 \quad \longrightarrow \quad A_x = 0$$

$$\sum F_y = 0$$

$$-A_y + 600 + B_y = 0$$

$$A_y = 600 + 1200 = 1800 \text{ N}$$

CHEK:

$$\sum M_C = 0$$

$$-1800 \cdot 2,4 + A_y \cdot 2,4 + A_x \cdot 2,7 = 0$$

$$-4320 + 1800 \cdot 2,4 + 0 \cdot 2,7 = 0$$

$$-4320 + 4320 = 0 \text{ (cocok)}$$

Bagian BCD:

$$\sum F_x = 0$$

$$B_x - C_x = 0$$

$$0 - C_x = 0 \quad \longrightarrow \quad C_x = 0$$

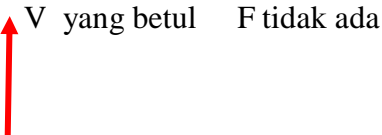
atau


$$\sum M_F = 0$$

$$A_y \cdot 4,8 + A_x \cdot 5,4 - C_x \cdot 2,7 - C_y \cdot 2,4 = 0$$

$$1800 \cdot 4,8 + 0 \cdot 5,4 - 0 \cdot 2,7 - 3600 \cdot 2,4 = 0 \text{ ---}$$

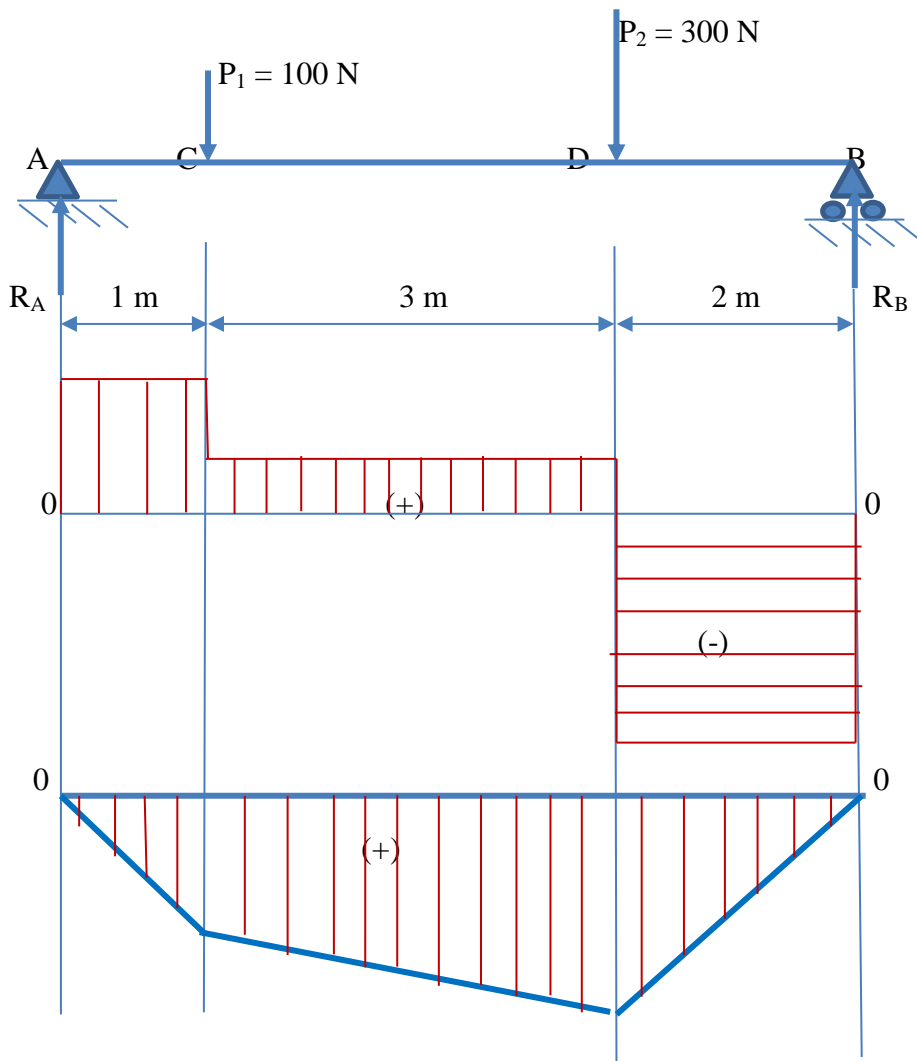
$$8640 - 0 - 0 - 8640 = 0 \text{ (cocok)}$$



 $V = -1200 \text{ N}$

GAYA LINTANG & BIDANG MOMEN

A. BEBAN TITIK



$$\sum M_A = 0$$

$$- R_B * 6 + P_2 * 4 + P_1 * 1 = 0$$

$$- R_B * 6 + 300 * 4 + 100 * 1 = 0$$

$$R_B = \frac{1300}{6} = 216,66 \text{ N}$$

$$\sum M_B = 0$$

$$R_A * 6 + P_1 * 5 + P_2 * 2 = 0$$

$$- R_A * 6 + 100 * 5 + 300 * 2 = 0$$

$$R_A = \frac{1100}{6} = 183,34 \text{ N}$$

Gaya Lintang (*Shear Force Diagram*/Bidang D):

$$0 \leq x \leq 1$$

$$D_0 = R_A = 183,34 \text{ N}$$

$$D_1 = R_A - P_1 = 183,34 - 100 = 83,34 \text{ N}$$

$$1 \leq x \leq 4$$

$$D_1 = R_A - P_1 = 183,34 - 100 = 83,34 \text{ N}$$

$$D_4 = R_A - P_1 - P_2 = 183,34 - 100 - 300 = -216,66 \text{ N}$$

$$4 \leq x \leq 6$$

$$D_4 = R_A - P_1 - P_2 = 183,34 - 100 - 300 = -216,66 \text{ N}$$

$$D_6 = R_A - P_1 - P_2 + R_B = 183,34 - 100 - 300 + 216,66 = 0 \text{ N}$$

Bidang Momen (*Bending Moment Diagram*/Bidang M):

$$0 \leq x \leq 1$$

$$M_x = R_A * x$$

$$M_0 = R_A * 0 = 0$$

$$M_1 = R_A * 1 = 183,34 * 1 = 183,34 \text{ N}$$

$$1 \leq x \leq 4$$

$$M_x = R_A * x - P_1(x-1)$$

$$M_1 = 183,34 * 1 - 100 (1-1) = 183,34 \text{ N}$$

$$M_4 = 183,34 * 4 - 100 (4-1) = 433,36 \text{ N}$$

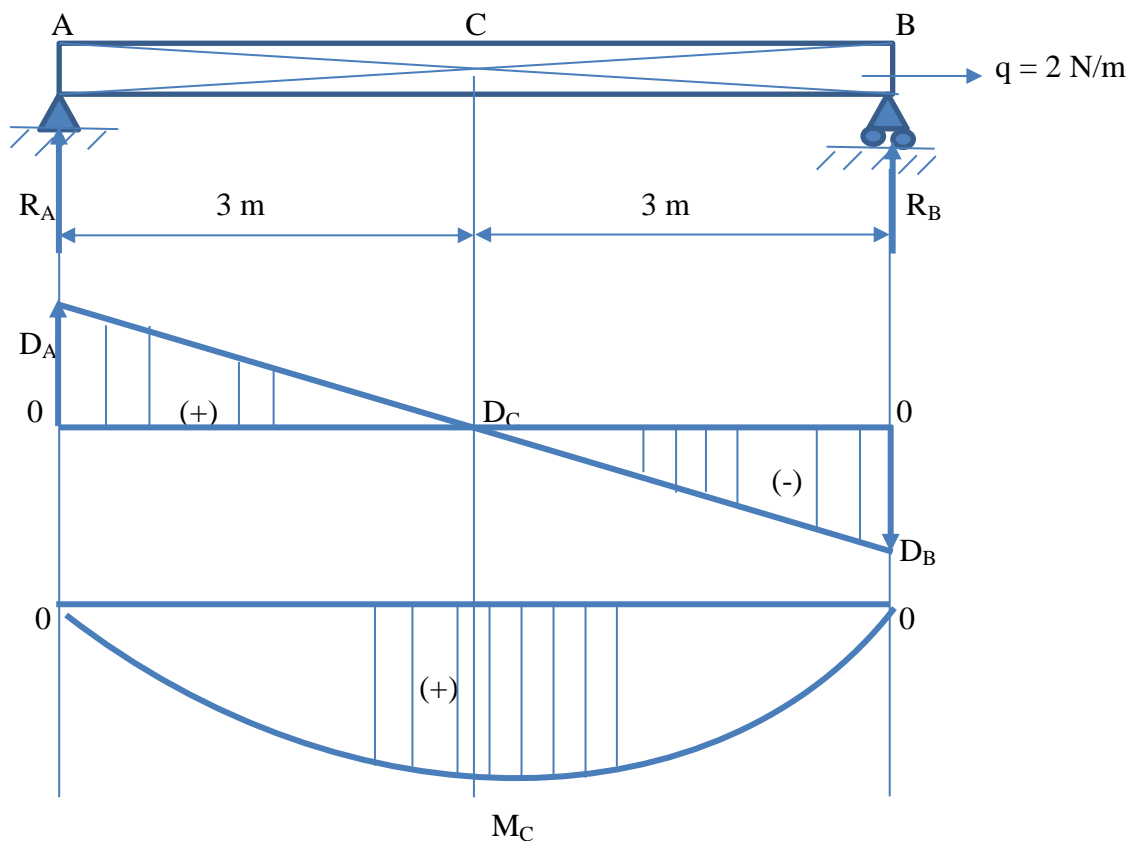
$$4 \leq x \leq 6$$

$$M_x = R_A * x - P_1(x-1) - P_2(x-4)$$

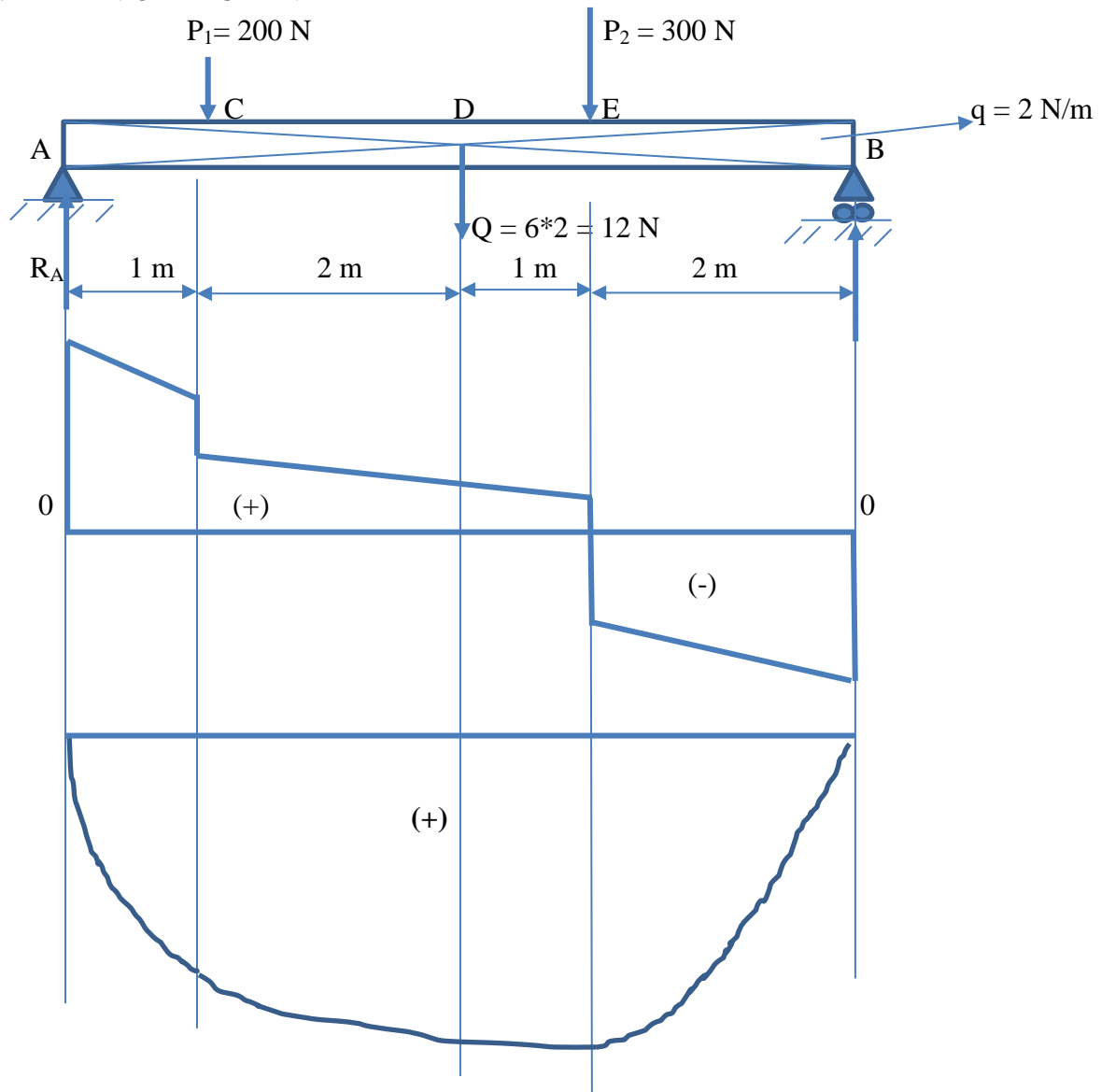
$$M_4 = 183,34 * 4 - 100 (4-1) = 433,36 \text{ N}$$

$$M_6 = 183,34 * 6 - 100 (6-1) - 300 (6-4) = 1100,4 - 500 - 600 = 0 \text{ N}$$

B. BEBAN MERATA



C. BEBAN CAMPURAN



$$\sum M_B = 0$$

$$R_A \cdot 6 - P_1 \cdot 5 - Q \cdot 3 - P_2 \cdot 2 = 0$$

$$R_A \cdot 6 - 200 \cdot 5 - 12 \cdot 3 - 300 \cdot 2 = 0$$

$$R_A \cdot 6 - 1000 - 36 - 600 = 0$$

$$R_A = \frac{1636}{6} = 272,67 \text{ N}$$

$$\sum M_A = 0$$

$$-R_B \cdot 6 + P_2 \cdot 4 + Q \cdot 3 + P_1 \cdot 1 = 0$$

$$-R_B \cdot 6 + 300 \cdot 4 + 12 \cdot 3 + 200 \cdot 1 = 0$$

$$-R_B \cdot 6 + 1200 + 36 + 200 = 0$$

$$R_B = \frac{1436}{6} = 239,33 \text{ N}$$

Gaya Lintang.

$$0 \leq x \leq 1$$

$$D_x = R_A - qx$$

$$D_x = 272,67 - 2x$$

$$x=0 : \quad D_A = 272,67 - 2*0 = 272,67 \text{ N}$$

$$x=1: \quad D_C = 272,67 - 2*1 = 270,67 \text{ N}$$

$$1 \leq x \leq 4$$

$$D_x = R_A - qx - P_1$$

$$D_x = 272,67 - 2x - 200$$

$$D_x = 72,67 - 2x$$

$$x=1: \quad D_C = 72,67 - 2*1 = 70,67 \text{ N}$$

$$x=4: \quad D_E = 72,67 - 2*4 = 64,67 \text{ N}$$

$$4 \leq x \leq 6$$

$$D_x = R_A - qx - P_1 - P_2$$

$$D_x = 272,67 - 2x - 200 - 300$$

$$D_x = -227,33 - 2x$$

$$x=4: \quad D_E = -227,33 - 2*4 = -235,33 \text{ N}$$

$$x=6: \quad D_B = -227,33 - 2*6 = -239,33 \text{ N}$$

Bidang Momen

$$0 \leq x \leq 1$$

$$M_x = R_A * x - \frac{1}{2}qx^2$$

$$M_x = 272,67*x - \frac{1}{2}2x^2$$

$$x=0 : \quad M_A = 272,67*0 - \frac{1}{2}*2*0^2 = 0 \text{ Nm}$$

$$x=1: \quad M_C = 272,67*1 - \frac{1}{2}*2*1^2 = 271,67 \text{ Nm}$$

$$1 \leq x \leq 4$$

$$M_x = R_A * x - \frac{1}{2}qx^2 - P_1(x-1)$$

$$M_x = 272,67*x - \frac{1}{2}2x^2 - 200(x-1)$$

$$M_x = 272,67*x - x^2 - 200x + 200$$

$$M_x = -x^2 + 72,67x + 200$$

$$x=1: \quad M_C = -1 + 72,67 + 200 = 271,67 \text{ Nm}$$

$$x=4: \quad M_E = -4^2 + 72,67*4 + 200 = -16 + 290,68 + 200 = 474,68 \text{ Nm}$$

$$4 \leq x \leq 6$$

$$M_x = R_A * x - \frac{1}{2} q x^2 - P_1(x-1) - P_2(x-4)$$

$$M_x = 272,67 * x - \frac{1}{2} 2 x^2 - 200 (x-1) - 300 (x-4)$$

$$M_x = 272,67 * x - x^2 - 200 x + 200 - 300 x + 1200$$

$$M_x = -x^2 - 227,33x + 1400$$

$$x=4: \quad M_E = -4^2 - 227,33*4 + 1400 = 474,68 \text{ Nm}$$

$$x=6: \quad M_B = -6^2 - 227,33*6 + 1400 = 0 \text{ Nm}$$

TITIK BERAT

Macam-macam titik berat:

- A. Titik berat garis
- B. Titik berat luas
- C. Titik berat volume

Untuk menentukan letak titik berat dilakukan dengan 2 cara:

1. Secara analitis

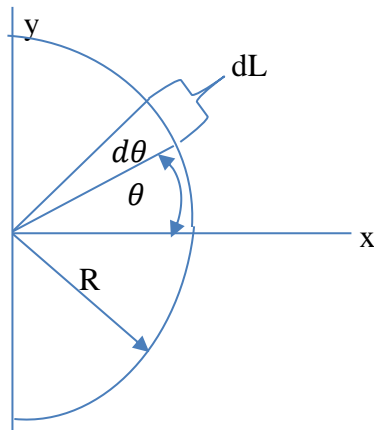
$$x = \frac{x * \sum A}{\sum A}$$

$$y = \frac{y * \sum A}{\sum y}$$

Contoh.

(1)

A. Titik berat garis



$$dL = R * d\theta$$

$$x = R \cdot \cos\theta$$

$$x = \frac{\int_{-\pi/2}^{\pi/2} x * dL}{\int_{-\pi/2}^{\pi/2} dL}$$

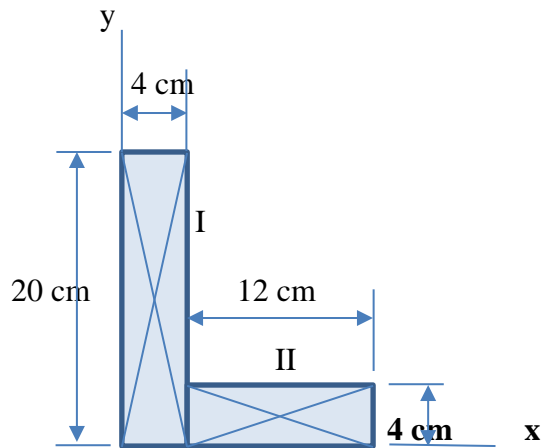
$$x = \frac{\int_{-\pi/2}^{\pi/2} R \cos\theta * R * d\theta}{\int_{-\pi/2}^{\pi/2} R * d\theta}$$

$$x = \frac{R^2 \sin\theta \Big|_{-\pi/2}^{\pi/2}}{R\theta \Big|_{-\pi/2}^{\pi/2}}$$

$$x = \frac{R^2(\sin \pi/2 + \sin \pi/2)}{R(\pi/2 + \pi/2)}$$

$$x = \frac{R^2 * 2}{R * \pi} = \frac{2R}{\pi} \quad (\text{dari titik pusat})$$

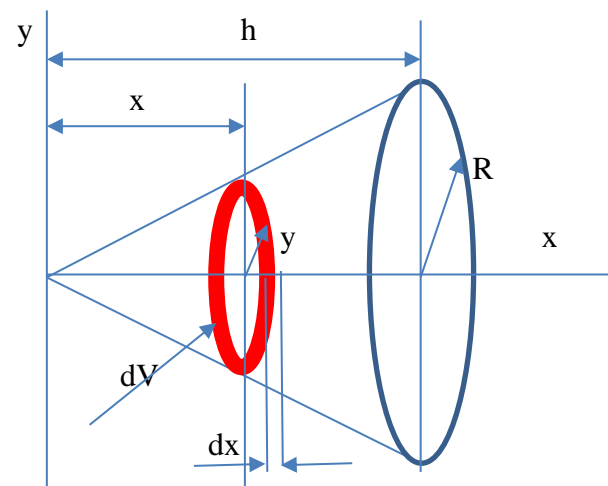
(2) Titik berat luas



$$x = \frac{x * \sum A}{\sum A} = \frac{(x_1 * 20 * 4) + (x_2 * 12 * 4)}{(20 * 4) + (12 * 4)} = \frac{(10 * 80) + (2 * 48)}{128} = \frac{896}{128} = 7 \text{ cm}$$

$$y = \frac{y * \sum A}{\sum A} = \frac{(y_1 * 20 * 4) + (y_2 * 12 * 4)}{(20 * 4) + (12 * 4)} = \frac{(2 * 80) + (10 * 48)}{128} = \frac{640}{128} = 5 \text{ cm}$$

B. Titik berat Volume



$$y : R = x : h$$

$$y = \frac{R * x}{h}$$

$$dV = \pi * y^2 * dx$$

$$x = \frac{\int_0^h x \cdot dV}{\int_0^h dV}$$

$$x = \frac{\int_0^h x \cdot \pi y^2 dx}{\int_0^h \pi y^2 dx}$$

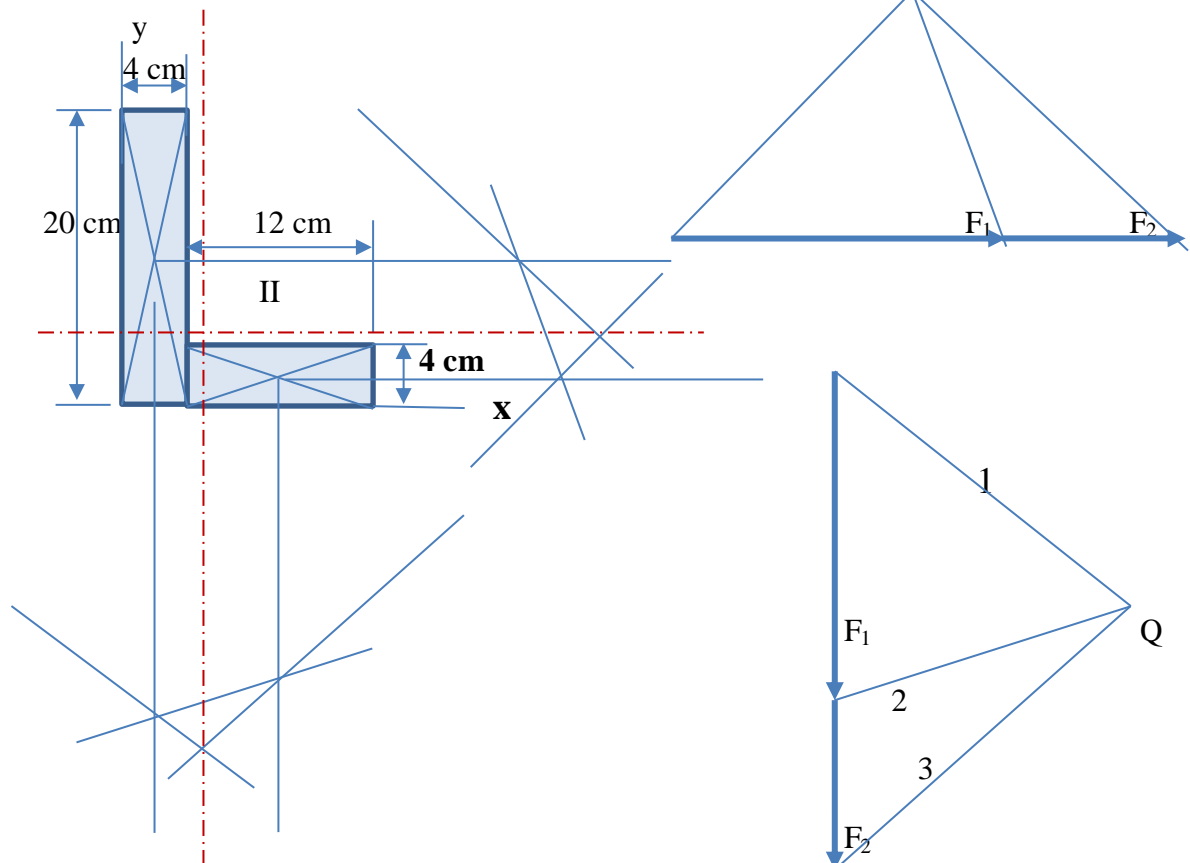
$$x = \frac{\int_0^h \pi \cdot x \cdot \frac{R^2 \cdot x^2}{h^2} dx}{\int_0^h \pi \frac{R^2 \cdot x^2}{h^2} dx}$$

$$x = \frac{\int_0^h \pi \frac{R^2 x^3}{h^2} dx}{\int_0^h \pi \frac{R^2 x^2}{h^2} dx}$$

$$x = \frac{\frac{\pi R^2}{h^2} \frac{1}{4} x^4 \Big|_0^h}{\frac{\pi R^2}{h^2} \frac{1}{3} x^3 \Big|_0^h}$$

$$x = \frac{\frac{1}{4} h^4}{\frac{1}{3} h^3} = \frac{3}{4} h \quad (\text{dari puncak})$$

2. Secara Grafis



MOMEN INERSIA (MOMEN LEMBAM)

Momen lembam adalah luas dikalikan dengan kwadrat jaraknya.

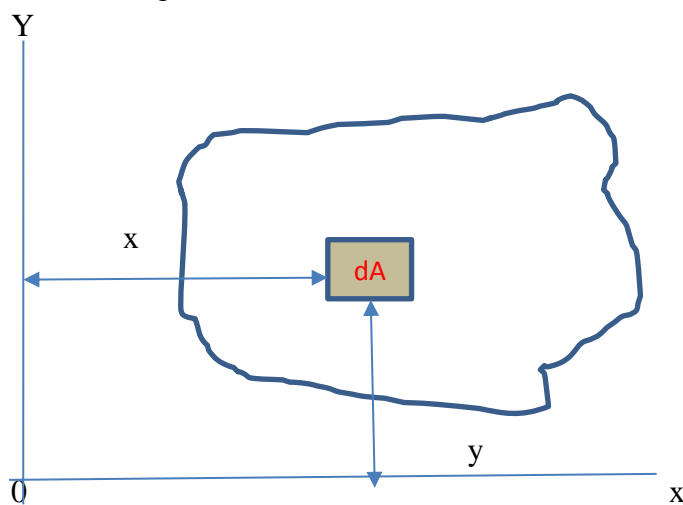
Momen lembam dibagi 2 yaitu:

- Momen lembam linier
- Momen lembam polair

- Momen Lembam linier.

Adalah luas penampang dikalikan dengan kwadrat jaraknya darinsuatu garis yang kita pandang.

Perhatikan gambar berikut ini:



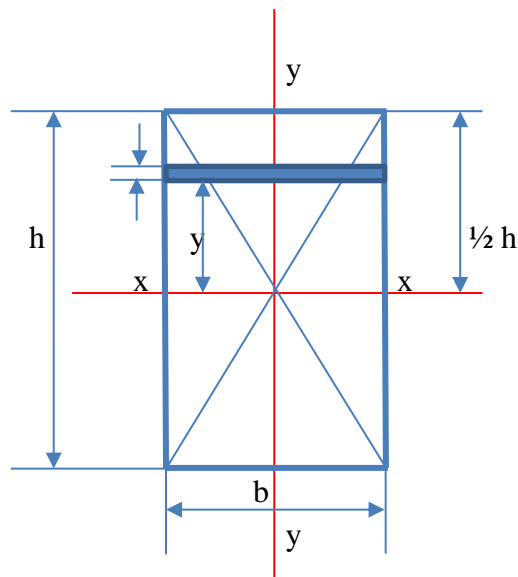
Momen lembam terhadap sumbu x (I_x):

$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$

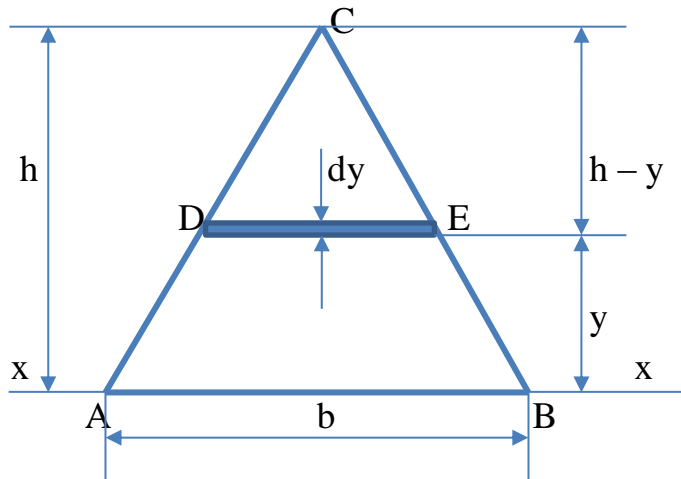
Contoh:

1.



$$\begin{aligned}
 I_x &= \int_{-\frac{1}{2}h}^{\frac{1}{2}h} y^2 dA \quad \text{dimana } dA = b \cdot dy \\
 &= \int_{-\frac{1}{2}h}^{\frac{1}{2}h} y^2 b \cdot dy \\
 &= b \int_{-\frac{1}{2}h}^{\frac{1}{2}h} y^2 dy \\
 &= b \left[\frac{1}{3} y^3 \right]_{-\frac{1}{2}h}^{\frac{1}{2}h} \\
 &= b \left[\frac{1}{3} \left(\frac{1}{2}h \right)^3 - \left(-\frac{1}{2}h \right)^3 \right] \\
 &= \frac{1}{3} b \left(\frac{1}{8}h^3 + \frac{1}{8}h^3 \right) \\
 &= \frac{1}{3} b \left(\frac{1}{4}h^3 \right) \\
 &= \frac{1}{12} bh^3 \quad \text{analog akan didapat } I_y = \frac{1}{12} hb^3
 \end{aligned}$$

2.



$$DE : AB = (h - y) : h$$

$$DE : b = (h - y) : h$$

$$DE = \frac{(h-y)}{h} b$$

$$dA = dE \cdot dy$$

$$\begin{aligned}
 I_x &= \int_0^h y^2 dA = \int_0^h y^2 \frac{(h-y)b}{h} dy = \int_0^h \frac{(bhy^2 - by^3)}{h} dy = \int_0^h \frac{bhy^2}{h} dy - \int_0^h \frac{by^3}{h} dy \\
 &= \int_0^h by^2 dy - \int_0^h \frac{b}{h} y^3 dy = \left[\frac{1}{3} by^3 \right]_0^h - \left[\frac{1}{4} \frac{b}{h} y^4 \right]_0^h \\
 &= \frac{1}{3} bh^3 - \frac{b}{h} \frac{1}{4} h^4 \\
 &= \frac{1}{3} bh^3 - \frac{1}{4} bh^3 \\
 &= \frac{1}{12} bh^3
 \end{aligned}$$

Jari-jari Girasi:

$$I = \int R^2 dA$$

$$I = R^2 A$$

$$R = \sqrt{\frac{I}{A}}$$

R : jari-jari girasi (kx)

$$I_x = \int kx^2 dA$$

$$I_x = kx^2 dA$$

Atau:

$$I_y = \int ky^2 dA$$

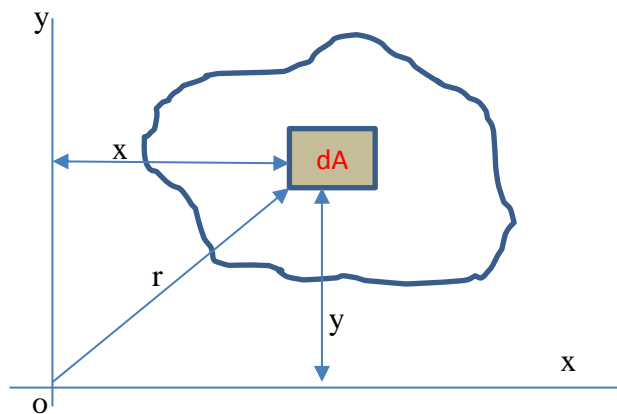
$$I_y = ky^2 dA$$

b. Momen Lembam Polair

Adalah momen lembam yang ditinjau dari titik perpotongan sumbu x dan sumbu y

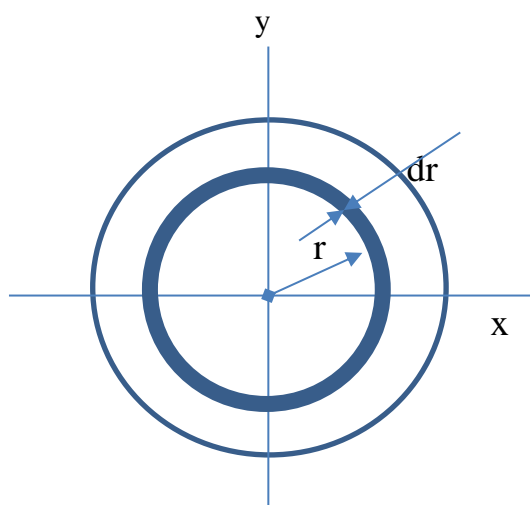
Jadi $I_p = I_x + I_y$

Perhatikan gambar dibawah ini.



$$I_p = \int r^2 dA$$

Contoh:



$$dA = 2\pi r \cdot dr$$

$$I_p = \int_0^{\frac{1}{2}d} r^2 dA$$

$$I_p = \int_0^{\frac{1}{2}d} r^2 \cdot 2\pi r \cdot dr$$

$$I_p = \int_0^{\frac{1}{2}d} r^3 \cdot 2\pi \cdot dr$$

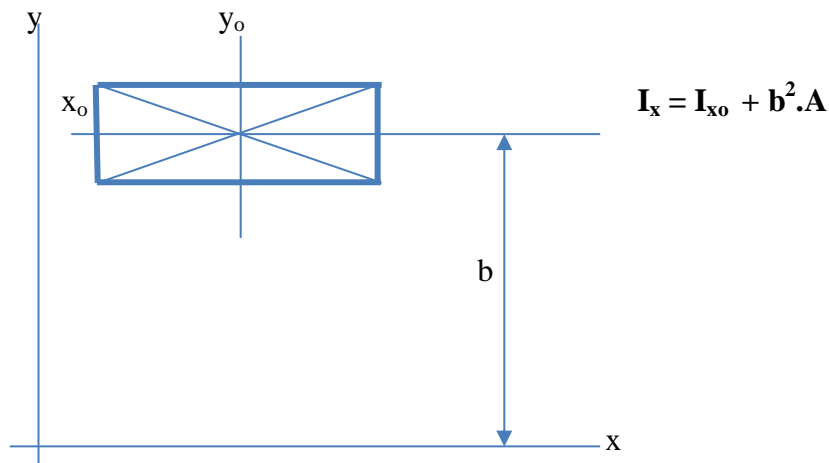
$$I_p = 2\pi \cdot \frac{1}{4} r^4 \Big|_0^{\frac{1}{2}d}$$

$$I_p = \frac{2}{4} \pi \left(\frac{1}{2} d \right)^4$$

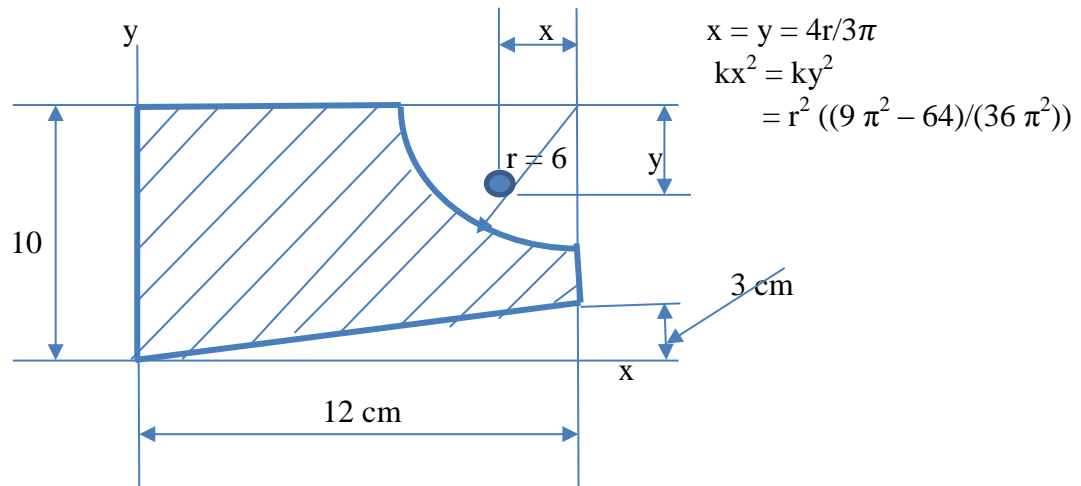
$$I_p = \frac{2}{4} \pi \frac{1}{16} d^4$$

$$I_p = \frac{\pi}{32} d^4$$

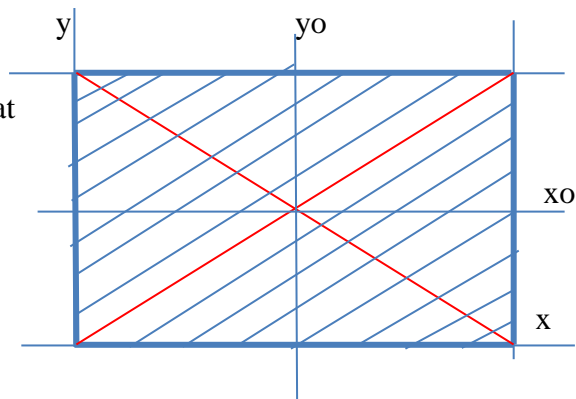
Teori Sumbu Sejajar:



Contoh:

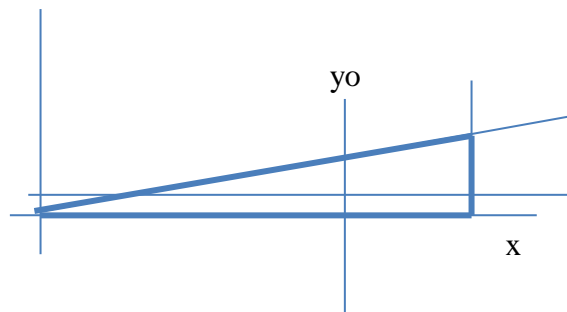


A. Segi empat



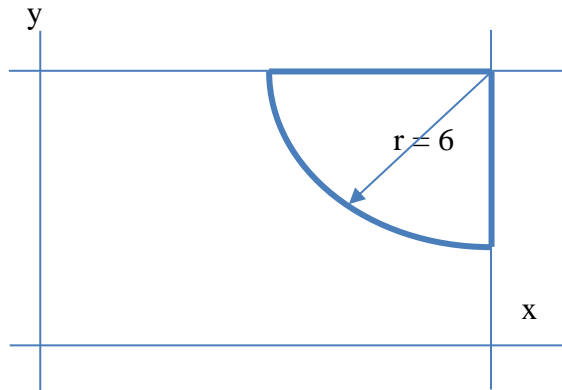
$$\begin{aligned}
 I_y &= I_{y_0} + b^2 A \\
 &= \frac{1}{12} b h^3 + 6^2 \cdot 10 \cdot 12 \\
 &= \frac{1}{12} \cdot 10 \cdot 12^3 + 36 \cdot 10 \cdot 12 \\
 &= \mathbf{5760 \text{ cm}^4}
 \end{aligned}$$

B. Segitiga



$$\begin{aligned}
 I_y &= I_{y_0} + b^2 A \\
 &= \frac{1}{36} b h^3 + b^2 A \\
 &= \frac{1}{36} \cdot 3 \cdot 12^3 + 8^2 \cdot \left(\frac{3 \cdot 12}{2} \right) \\
 x_0 &= \mathbf{1296 \text{ cm}^4}
 \end{aligned}$$

C. Seperempat Lingkaran



$$\begin{aligned}
 I_y &= I_{y_0} + b^2 A \\
 &= k y^2 \cdot A + b^2 \cdot A \\
 &= \left(r^2 \frac{9\pi^2 - 64}{36 \cdot \pi^2} \right) \left(\frac{\pi}{4} r^2 \right) + \left(12 - \frac{4r}{3\pi} \right)^2 \left(\frac{\pi}{4} r^2 \right) \\
 &= \left(6^2 \frac{9\pi^2 - 64}{36 \cdot \pi^2} \right) \left(\frac{\pi}{4} 6^2 \right) + \left(12 - \frac{4 \cdot 6}{3\pi} \right)^2 \left(\frac{\pi}{4} 6^2 \right) \\
 &= 70,65 + 2529 \\
 &= \mathbf{2599,65 \text{ cm}^4}
 \end{aligned}$$

$$\begin{aligned}
 \text{Jadi } I_y &= A - B - C \\
 &= 5760 \text{ cm}^4 - 1296 \text{ cm}^4 - 2599,6 \text{ cm}^4 \\
 &= \mathbf{1864,4 \text{ cm}^4}
 \end{aligned}$$