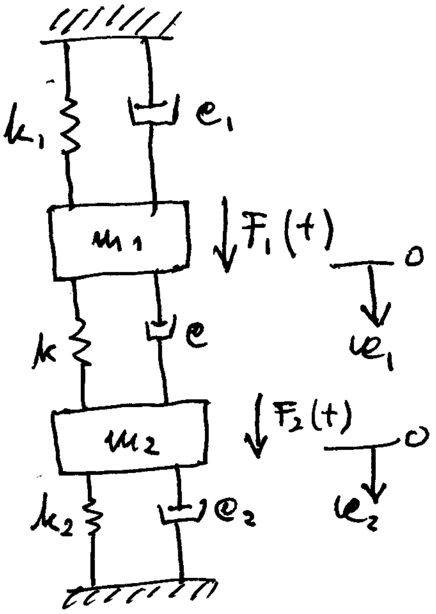
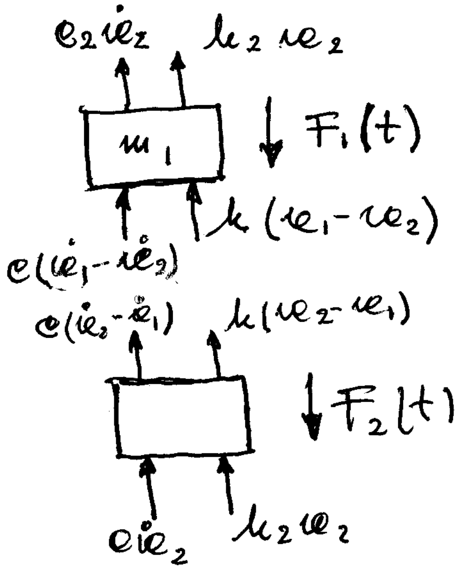
Dua Derajat Kebebasan





Dengan menggunakan hukum Newton II :

$$m_1\ddot{x}_1 = -k_1x_1 - k(x_1-x_2) - c_1\dot{x}_1 - c(\dot{x}_1-\dot{x}_2) + F_1(t)$$
$$m_2\ddot{x}_2 = -k_2x_2 - k(x_2-x_1) - c_2\dot{x}_2 - c(\dot{x}_2-\dot{x}_1) + F_2(t)$$

Persamaan diatas disusun kembali menjadi:

$$m_1\ddot{x}_1 + (c_1+c)\dot{x}_1 + (k_1+k)x_1 - c\dot{x}_2 - kx_2 = F_1(t)$$
$$m_2\ddot{x}_2 + (c_2+c)\dot{x}_2 + (k_2+k)x_2 - c\dot{x}_1 - kx_1 = F_2(t)$$

$F_1(t)$ & $F_2(t_2)$: gaya eksitasi thd m_1 & m_2.

Dari persamaan diatas terdapat :

$-(c\dot{x}_2 + kx_2)$ dan

$-(c\dot{x}_1 + kx_1)$

Dengan kata lain bahwa gerak massa 1, $x_1(t)$ dipengaruhi gerak massa 2, $x_2(t)$ dan sebaliknya.

Dalam bentuk matriks ditulis :

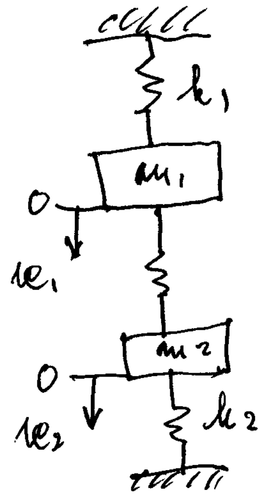
$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1+c & -c \\ -c & c_2+c \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k & -k \\ -k & k_2+k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix}$$

atau

$$[M](\ddot{q}) + [C](\dot{q}) + [K](q) = \{Q(t)\}$$

$[M]$: matriks massa ; $[C]$: matriks redaman ; $[K]$: matriks kekakuan
(x): matriks kedudukan ; $[F(t)]$: matrik gaya eksitasi

Getaran bebas tidak teredam



Persamaan geraknya adalah

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1+k & -k \\ -k & k_2+k \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{---} (1)$$

Persamaan diatas adalah linier & homogen

$u_1 = B_1 e^{st}$

$u_2 = B_2 e^{st}$

B_1, B_2 dan s adalah konstanta. Karena sistem tersebut tidak teredam, nilai s adalah imajiner $s = \pm jw$. Dengan menggunakan Euller $e^{jwt} = \cos wt \pm j \sin wt$, maka solusi persamaan diatas adalah harmonik:

$$u_1 = A_1 \sin(wt + \psi) \quad \text{----} (2)$$
$$u_2 = A_2 \sin(wt + \psi)$$

A_1, A_2 dan ψ adalah konstanta, w : frekuensi pribadi

Subetitusi (2) ke (1) dan dibagi $\sin(wt + \psi)$ maka diperoleh:

$$(k + k_1 - w^2 m_1) A_1 - k A_2 = 0 \quad \text{----} (3)$$
$$-k A_1 + (k + k_2 - w^2 m_2) A_2 = 0$$

Determinan $\Delta(w)$ dg koefisien A_1 & A_2 disebut determinan karakteristik. Jika $\Delta(w) = 0$ maka akan diperoleh pers. frekuensi sistem dan akan didapat frekuensi pribadi sistem.

$$\Delta(w) = \begin{vmatrix} k + k_1 - w^2 m_1 & -k \\ -k & k + k_2 - w^2 m_2 \end{vmatrix} = 0 \quad \text{----} (A)$$

$$w^4 - \left(\frac{k + k_1}{m}\right) w^2 + \frac{k_1 k_2 + k_1 k + k_2 k}{m_1 m_2} = 0 \quad \text{---} (B)$$

kurang

Akan didapat w_1^2 dan w_2^2

Dengan cara yg sama, pers (2) solusinya

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} A_{1\cdot1} \\ A_{2\cdot1} \end{bmatrix} \sin(\omega_1 t + \psi_1) + \begin{bmatrix} A_{1\cdot2} \\ A_{2\cdot2} \end{bmatrix} \sin(\omega_2 t + \psi_2) \quad \cdots (5)$$

$A_{1\cdot2}$ adalah amplitudo $x_1(t)$ pada frekuensi $\omega = \omega_2$

Amplitudo relatif komponen harmonik (5) diperoleh dg mensubtitusikan ω_1 & ω_2 pada pers (3) maka:

$$\frac{A_{11}}{A_{21}} = \frac{k}{k+k_1-\omega_1^2 m_1} = \frac{k+k_2-\omega_1^2 m_2}{k} = \frac{u_{11}}{u_{21}} = \frac{1}{u_1}$$

$$\frac{A_{12}}{A_{22}} = \frac{k}{k+k_1-\omega_1^2 m_1} = \frac{k+k_2-\omega_2^2 m_2}{k} = \frac{u_{1\cdot2}}{u_{22}} = \frac{1}{u_2} \quad \cdots (6)$$

Pers (5) menjadi:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ u_1 \end{bmatrix} A_{11} \sin(\omega_1 t + \psi_1) + \begin{bmatrix} 1 \\ u_2 \end{bmatrix} A_{12} \sin(\omega_2 t + \psi_2) \quad \cdots (7)$$

A_{11}, A_{12}, ψ_1 dan ψ_2 adalah konstanta integrasi yg diperoleh dari kondisi awal.

Jika $A_{12} = 0$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ u_1 \end{bmatrix} A_{11} \sin(\omega_1 t + \psi_1) \text{ atau } \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} u_{11} \\ u_{21} \end{bmatrix} p_1(t) = (u)_1 p_1(t)$$

$(u)_1$ = vektor Eigen pada mode pertama = menunjukkan amplitudo relatif atau mode getar $x_1(t)$ dan $x_2(t)$ pada $\omega = \omega_1$

Jika $A_{1\cdot1} = 0$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ u_2 \end{bmatrix} A_{12} \sin(\omega_2 t + \psi_2) \text{ atau } \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} u_{1\cdot2} \\ u_{22} \end{bmatrix} p_2(t) = (u)_2 \cdot p_2(t)$$

Fungsi harmonik $x_1(t)$ dan $x_2(t)$ pers (7):

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ u_1 & u_2 \end{bmatrix} \begin{bmatrix} A_{11} \sin(\omega_1 t + \psi_1) \\ A_{12} \sin(\omega_2 t + \psi_2) \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} p_1(t) \\ p_2(t) \end{bmatrix} \quad \cdots (8)$$

atau $(x) = (u)(p)$.

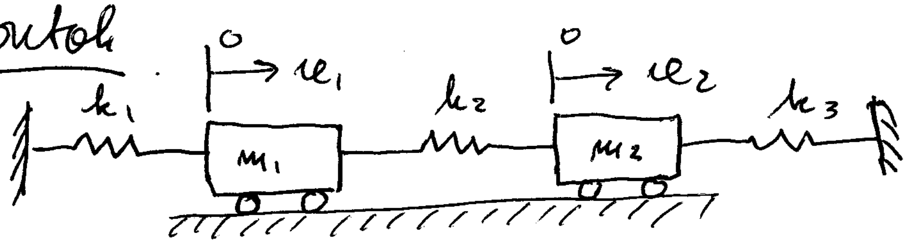
dimana matriks modus getar (u) :

$$[u] = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ u_1 & u_2 \end{bmatrix}$$

atau

$$[u] = [u_{ij}] = [(u)_1 \quad (u)_2]$$

Contoh



$m_1 = m_2 = m$
$k_1 = k_2 = k$
$u_1(0) = 1$; $\dot{u}_1(0) = 1$
$u_2(0) = 0$; $\dot{u}_2(0) = 0$

Ditanyakan
a. Frekuensi pribadi sistem
b. Vektor perpindahan (u)

Dari pers. (4) didapat

$$\omega^4 - \frac{4k}{m}\omega^2 + \frac{3k^2}{m^2} = 0$$

Dengan rumus abc :

$$\omega_{1.2}^2 = \frac{4k}{2m} \pm \frac{1}{2}\sqrt{\left(\frac{4k}{m}\right)^2 - 4\left(\frac{3k^2}{m^2}\right)}$$

$$\omega_{1.2}^2 = \frac{2k}{m} \pm \frac{1}{2m}\sqrt{16k^2 - 12k^2} = \frac{2k}{m} \pm \frac{k}{m}.$$

$$\omega_1^2 = \frac{k}{m} \; ; \; \omega_2^2 = \frac{3k}{m}$$

$$\therefore \; \omega_1 = \sqrt{\frac{k}{m}} \qquad \omega_2 = \sqrt{\frac{3k}{m}}.$$

Substitusikan ke pers (6)

$$\frac{1}{u_1} = \frac{k+k_2-\omega_1^2 m_2}{k} = \frac{2k-(k/m)m}{k} = 1$$

$$\frac{1}{u_2} = \frac{2k-(3k/m)m}{k} = -1$$

Vektor perpindahan u menggunakan pers (8)

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ u_1 & u_2 \end{bmatrix} \begin{bmatrix} A_{1.1}\sin(\omega_1 t + \varphi_1) \\ A_{12}\sin(\omega_2 t + \varphi_2) \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} A_{11}\sin(\omega_1 t + \varphi_1) \\ A_{12}\sin(\omega_2 t + \varphi_2) \end{bmatrix} \rightarrow \text{masukan kondisi awal } \{u(0)\} = \{1 \;\; 0\}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} A_{11}\sin\varphi_1 \\ A_{12}\sin\varphi_2 \end{bmatrix}$$

$$1 = A_{11}\sin\varphi_1 + A_{12}\sin\varphi_2$$
$$\underline{0 = A_{11}\sin\varphi_1 - A_{12}\sin\varphi_2} \;\; +$$
$$1 = 2A_{11}\sin\varphi_1$$

$\rightarrow A_{11} = \dfrac{1}{2\sin\varphi_1}$; analog

$A_{12} = \dfrac{1}{2\sin\varphi_2}$

Dengan menggunakan kondisi awal kedua $\{\dot{x}(0)\} = \{0\ 0\}$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \omega_1 A_{11} \cos \varphi_1 \\ \omega_2 A_{12} \cos \varphi_2 \end{bmatrix}$$

$$0 = \omega_1 A_{11} \cos \varphi_1 + \omega_2 A_{12} \cos \varphi_2$$
$$0 = \omega_1 A_{11} \cos \varphi_1 - \omega_2 A_{12} \cos \varphi_2 \quad +$$
$$0 = 2\omega_1 A_{11} \cos \varphi_1$$

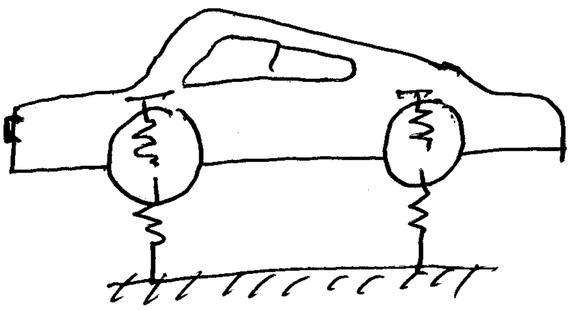
Karena A_{11} & $\omega_1 \neq 0$, maka $\cos \varphi_1 = 0$.

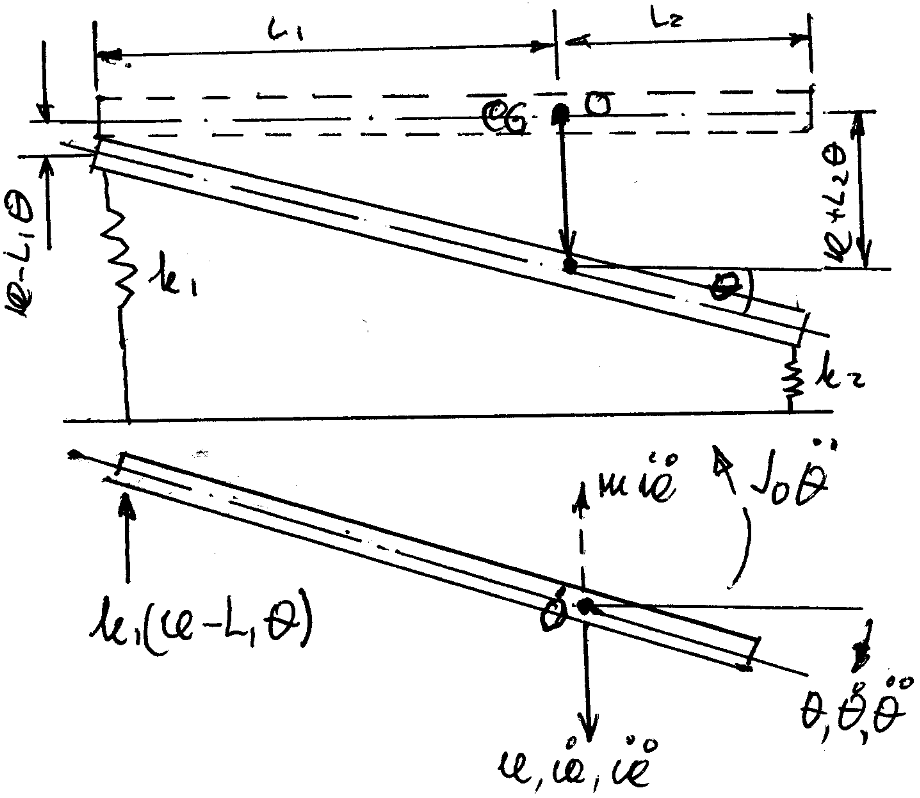
Analog $\cos \varphi_2 = 0$, maka :

$$\varphi_1 = \varphi_2 = (n+1)\frac{\pi}{2}, \quad n = 0, 1, 2 \ldots$$

$$A_{11} = A_{12} = \frac{1}{2}.$$

Suspensi Kendaraan





Persamaan gerak dalam koordinat $x(t)$ dan $\theta(t)$:

$$m\ddot{x} = \Sigma(\text{Gaya})$$

$$m\ddot{x} = -k_1(x - L_1\theta) - k_2(x + L_2\theta)$$

$$m\ddot{x} + k_1(x - L_1\theta) + k_2(x + L_2\theta) = 0$$

$$m\ddot{x} + (k_1 + k_2)x - (k_1L_1 - k_2L_2)\theta = 0 \quad \text{--- (a)}$$

dan:

$$J_0\ddot{\theta} = \Sigma(\text{momen})$$

$$J_0\ddot{\theta} = k_1(x - L_1\theta)L_1 - k_2(x - L_2\theta)L_2$$

$$J_0\ddot{\theta} - (k_1L_1 - k_2L_2) + (k_1L_1^2 + k_2L_2^2)\theta = 0 \quad \text{--- (b)}$$

$$\begin{bmatrix} m & 0 \\ 0 & J_0 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -(k_1L_1 - k_2L_2) \\ -(k_1L_1 - k_2L_2 & k_1L_1^2 + k_2L_2^2 \end{bmatrix}\begin{Bmatrix} x \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Persamaan frekuensi sistem (lihat pers (A))

$$\Delta(\omega) = \begin{vmatrix} k_1+k_2-\omega^2 m & k_2L_2-k_1L_1 \\ k_2L_2-k_1L_1 & k_1L_1^2+k_2L_2^2 \end{vmatrix} = 0$$

Dengan menggunakan rumus a b c spt pad (A):

$$\omega_{1,2}^2 = \frac{1}{2}\left[\frac{k_1+k_2}{m} + \frac{k_1L_1^2+k_2L_2^2}{J_0} \pm \sqrt{\left(\frac{k_1+k_2}{m}\right) + \frac{k_1L_1^2+k_2L_2^2}{J_0} - \frac{4k_1k_2(L_1+L_2)^2}{mJ_0}}\right]$$

Contoh:

Sebuah mobil dg massa 2000 kg, jarak antar roda 3,5 m, pusat massa terletak 1,5 m dari roda depan. Radius girasi 1,4 m. Konstante pegas depan & belakang 40 kN/m dan 50 kN/m.

Hitung: a. Frekuensi sistem.
b. Mode getar
c. Gerak $x(t)$ dan $\theta(t)$.

Jawab:

$$\frac{k_1+k_2}{m} = \frac{40\,000\ N/m + 50\,000\ N/m}{2000\ kg} = 45$$

$$J_0 = m \cdot r_G^2 = (2000\ kg)(1,4\ m)^2 = 3920\ kg\,m^2$$

$$\frac{k_1L_1^2+k_2L_2^2}{J_0} = \frac{(40000)(1,5)^2+(50000)(2)^2}{3920} = 74$$

$$\frac{k_1L_1-k_2L_2}{m} = \frac{(40000)(1,5)-(50000)(2)}{2000} = -20$$

$$\frac{4k_1k_2(L_1+L_2)^2}{mJ_0} = \frac{4(40000)(50000)(1,5+2)^2}{(2000)(3920)} = 12500$$

(a) Frekuensi pribadi sistem:

$$\omega_{12}^2 = \frac{1}{2}\left[45+74 \pm \sqrt{(45+74)^2 - 12500}\right] = \begin{cases}18,745\\ 100,255\end{cases}$$

$$\omega_{1,2} = \begin{cases}\sqrt{18,745} = 4,33 \text{ rad/s}\\ \sqrt{100,255} = 10,01 \text{ rad/s.}\end{cases}$$

(b) Rasio Amplitudo:

$$\frac{X}{\theta} = \frac{k_1 l_1 - k_2 l_2}{k_1 + k_2 - \omega^2 m} \quad \text{atau} \quad \frac{X}{\theta} = \frac{(k_1 L_1 - k_2 L_2)/m}{(k_1+k_2)/m - \omega_{1,2}^2}$$

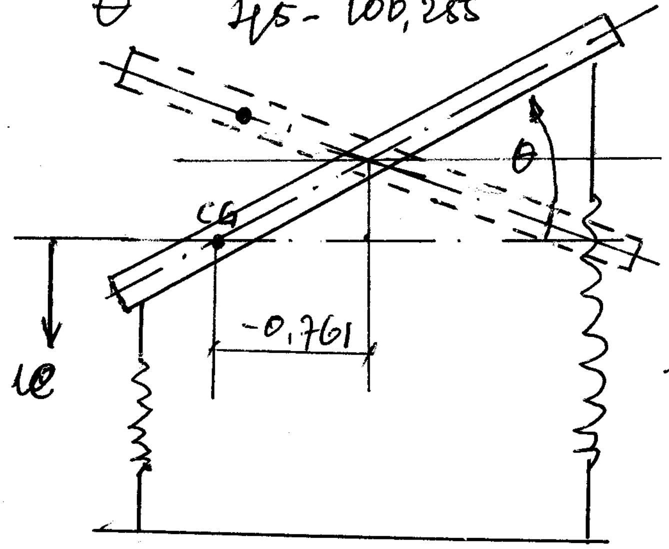
Modus getar didapat dari rasio amplitudo:

Modus pertama:

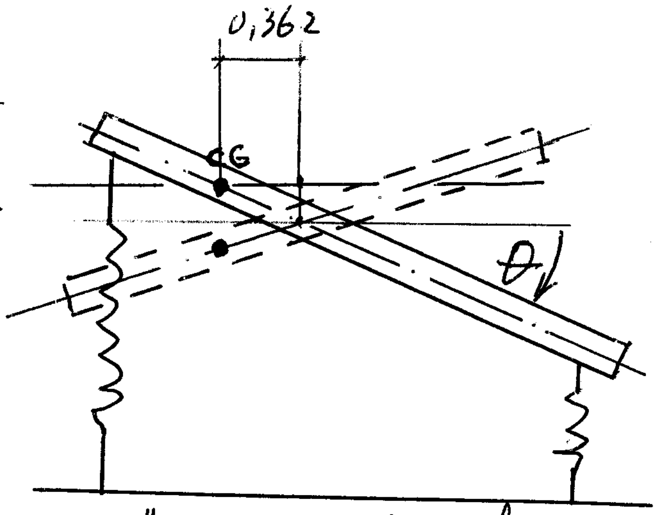
$$\frac{X}{\theta} = \frac{-20}{45 - 18,745} = -0,761$$

Modus getar kedua:

$$\frac{X}{\theta} = \frac{-20}{45 - 100,255} = 0,362.$$



a. Modus getar pertama



b. Modus getar kedua

(c) Gerak $x(t)$ dan $\theta(t)$ didapat:

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 \\ -1/0{,}761 \end{bmatrix} A_{11} \sin(\omega_1 t + \varphi_1) + \begin{bmatrix} 1 \\ 1/0{,}362 \end{bmatrix} A_{12} \sin(\omega_2 t + \varphi_2)$$

$$x = A_{11} \sin(\omega_1 t + \varphi_1) + A_{12} \sin(\omega_2 t + \varphi_2)$$

$$\theta = -\frac{1}{0{,}761} A_{11} \sin(\omega_1 t + \varphi_1) + \frac{1}{0{,}362} A_{12} \sin(\omega_2 t + \varphi_2)$$

Atau:

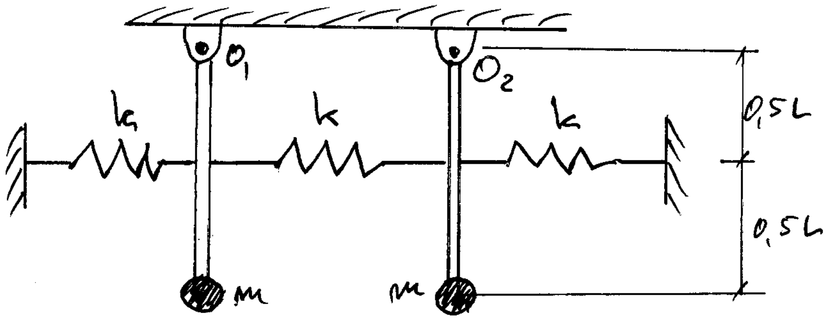
$$Q = \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1{,}3141 & 2{,}7624 \end{bmatrix} \begin{bmatrix} A_{11} \sin(\omega_1 t + \varphi_1) \\ A_{12} \sin(\omega_2 t + \varphi_2) \end{bmatrix}$$

Kecepatan $\dot{x}(t)$ dan $\dot{\theta}(t)$:

$$\dot{Q} = \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1{,}3141 & 2{,}7624 \end{bmatrix} \begin{bmatrix} \omega_1 A_{11} \cos(\omega_1 t + \varphi_1) \\ \omega_2 A_{12} \cos(\omega_2 t + \varphi_2) \end{bmatrix}$$

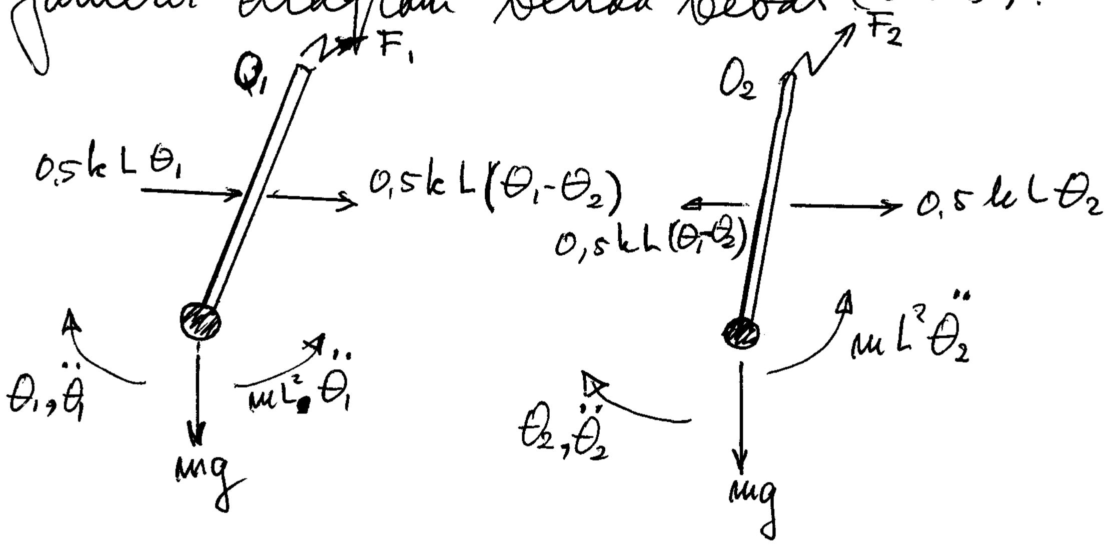
A_{11}; A_{12}; φ_1; φ_2 = konstante integrasi.

Seperti pada gambar dibawah ini. Turunkan persamaan gerak dan frekuensi natural.



Jawab.

Gambar diagram benda bebas (DBB):



DBB massa 1:

$$J_{O_1}\cdot\ddot{\theta}_1 = \sum(\text{Momen})O_1$$

$$mL^2\cdot\ddot{\theta}_1 = -(0{,}5kL\theta_1)(0{,}5L)-(0{,}5kL(\theta_1-\theta_2))(0{,}5L)-mgL\theta_1$$

$$mL^2\ddot{\theta}_1+(0{,}5kL^2+mgL)\theta_1-0{,}25kL^2\theta_2=0 \quad \text{- - - -} \; (1)$$

DBB massa 2:

$$J_{O_2}\cdot\ddot{\theta}_2 = \sum(\text{Momen})O_2$$

$$mL^2\cdot\ddot{\theta}_2 = -(0{,}5kL\theta_2)(0{,}5L)+(0{,}5kL(\theta_1-\theta_2))(0{,}5L)-mgL\theta_2 \quad \text{- - - -} \; (2)$$

$$mL^2\ddot{\theta}_2+(0{,}5kL^2+mgL)\theta_2-0{,}25kL^2\theta_1=0$$

Dari (1) & (2) disusun dlm bentuk matrik.

$$\begin{bmatrix} mL^2 & 0 \\ 0 & mL^2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}+\begin{bmatrix} 0{,}5kL^2+mgL & -0{,}25kL^2 \\ -0{,}25kL^2 & 0{,}5kL^2+mgL \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \cdots (3)$$

$\ddot{\theta}_1 = -\omega^2 \Theta_1$ dan $\ddot{\theta}_2 = \omega^2 \Theta$, substitusikan ke (3)

maka:

$$\begin{bmatrix} 0{,}5kL^2 + mgL - mL^2\omega^2 & -0{,}25kL^2 \\ 0{,}25kL^2 & 0{,}5kL^2 + mgL - mL^2\omega^2 \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \cdots (4)$$

Persamaan frekuensi:

$$\Delta(\omega) = \begin{vmatrix} 0{,}5k \cdot L^2 + mgL - mL^2\omega^2 & -0{,}25kL^2 \\ -0{,}25kL^2 & 0{,}5kL^2 + mgL - mL^2\omega^2 \end{vmatrix} = 0 \cdots (5)$$

Dengan menguraikan determinan (5):

$$(0{,}5k.L^2 + mgL - mL^2\omega^2)(0{,}5kL^2 + mgL - mL^2\omega^2) - (-0{,}25kL^2)(-0{,}25kL^2)$$

$$0{,}25k^2L^4 + 0{,}5kL^3mg - 0{,}5kL^4m\omega^2$$
$$+ 0{,}5mgkL^3 + m^2g^2L^2 - m^2gL^3\omega^2$$
$$- 0{,}5mL^4\omega^2k - m^2L^3\omega^2g + m^2L^4\omega^4 - 0{,}0625k^2L^4 = 0$$

Kemudian dibagi L^2:

$$0{,}25k^2L^2 + 0{,}5kLmg - 0{,}5kL^2m\omega^2$$
$$+ 0{,}5mgkL + m^2g^2 - m^2gL\omega^2 + m^2L^2\omega^4$$
$$- 0{,}5mL^2\omega^2k - m^2L\omega^2g - 0{,}0625k^2L^2 = 0$$

$$m^2L^2\omega^4 - kL^2m\omega^2 - 2m^2gL\omega^2 + mgkL + m^2g^2 + 0{,}1875k^2L^2 = 0$$

$$m^2L^2\omega^4 - (kL^2m + 2m^2gL)\omega^2 + m^2g^2 + mgkL + 0{,}1875k^2L^2 = 0 \quad \cdots (6)$$

Dengan menggunakan rumus A, B, C:

$A = m^2L^2$; $B = -(kL^2m + 2m^2gL)$;

$C = m^2g^2 + mgkL + 0{,}1875k^2L^2$

$$\omega_{1\cdot 2}^2 = \frac{-B \pm \sqrt{B^2 - 4 \cdot A \cdot C}}{2A}$$

Jika diketahui $m = 0{,}5$ kg ; $L = 0{,}2$ m ; $k = 100$ N/m
$g = 10$ m/s².

Hitung : a. Frekuensi pribadi.
b. Modus getar.

Jawab.

(a). $A = m^2 L^2 = (0{,}5\,kg)^2 (0{,}2\,m)^2 = \underline{0{,}01\,kg^2\,m^2}$

$B = -(kL^2 m + 2m^2 g L) = -\left[(100\,N/m)(0{,}2\,m)^2(0{,}5\,kg) + 2(0{,}5\,kg)^2(10\,\frac{m}{s^2})(0{,}2\,m)\right]$

$= -(2\ N.kg.m + 1\ kg^2.\frac{m}{s^2}.m)$

$= -(2\ kg.\frac{m}{s^2}.kg.m + 1\ kg^2.\frac{m}{s^2}.m)$

$= \underline{-3\ kg^2\ m^2/s^2}$

$C = m^2 g^2 + mgkL + 0{,}1875\,k^2.L^2$

$= \left[(0{,}5\,kg)^2 \cdot (10\,\frac{m}{s^2})^2\right] + \left[(0{,}5\,kg)(10\,\frac{m}{s^2})(100\,\frac{N}{m}) + (0{,}2\,m)\right] + \left[(0{,}1875)(100\,\frac{N}{m})^2 + (0{,}2\,m)^2\right]$

$= 25\ kg^2.\frac{m^2}{s^4} + 100\ kg\,\frac{m}{s^2}.\frac{N}{m}.m + 75\,\frac{N^2}{m^2}\,m^2.$

$= 25\ kg^2.\frac{m^2}{s^4} + 100\ kg\,\frac{m}{s^2}.\frac{kg.\frac{m}{s^2}}{\cancel{m}}.\cancel{m} + 75\,\frac{kg^2.\frac{m^2}{s^4}}{\cancel{m^2}}\cancel{m^2}$

$= \underline{200\ kg^2\ m^2/s^4}$

Jadi persamaan (6) menjadi:

$0{,}01\,\omega^4 - 3\,\omega^2 + 200 = 0.$

$\omega_{1.2}^2 = \frac{3 \pm \sqrt{9-8}}{0{,}02} = \frac{3 \pm 1}{0{,}02}$

$\omega_1^2 = \frac{3+1}{0{,}02} = 200$ rad/s

$\omega_2^2 = \frac{3-1}{0{,}02} = 100$ rad/s

b) Untuk memperoleh modus getar, dari pers (1) subtitusi dg $\ddot{\theta}_i = -\omega^2 \Theta_i$, maka pers. (1) menjadi

$$-mL^2 \cdot \omega^2 \Theta_1 + (0{,}5kL^2 + mgL)\Theta_1 - 0{,}25kL^2\Theta_2 = 0$$

$$\frac{\Theta_1}{\Theta_2} = \frac{0{,}25kL^2}{0{,}5kL^2 + mgL - \omega^2 mL^2} = \frac{1}{3 - 0{,}02\,\omega^2}$$

Untuk $\omega^2 = \omega_1^2 = 200$ rad/s

$$\frac{\Theta_1}{\Theta_2} = \frac{1}{3 - 0{,}02\,(200)} = -1$$

Untuk $\omega^2 = \omega_2^2 = 100$ rad/s

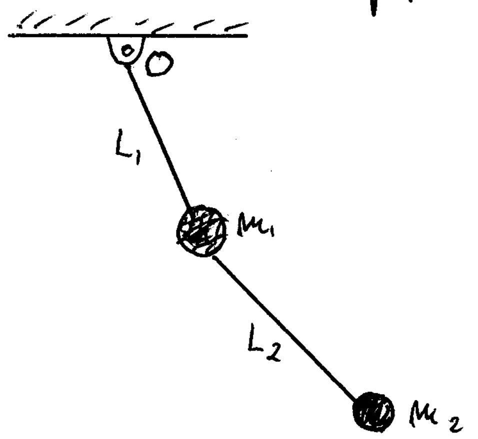
$$\frac{\Theta_1}{\Theta_2} = \frac{1}{3 - 0{,}02\,(100)} = 1$$

Vektor modus getar:

$$\Phi = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

1) Seperti gambar dibawah ini, massa batang L_1 & L_2 diabaikan.

Tentukan: a. Persamaan gerak sistem
b. Frekuensi natural.
c. Modus getar



Jawab

Percepatan massa m_1:

$$A_{m_1} = \ddot{\theta}_1 \cdot L_1$$

Percepatan massa m_2:

$$A_{m_2} = A_{m_1} + \rightarrow A_{m_2/m_1}$$

$$= \underset{\perp L_1}{\underline{\ddot{\theta}_1 L_1}} + \underset{\perp L_2}{\underline{\ddot{\theta}_2 L_2}}$$

Gaya inersia yg bekerja pada m_1 & m_2:

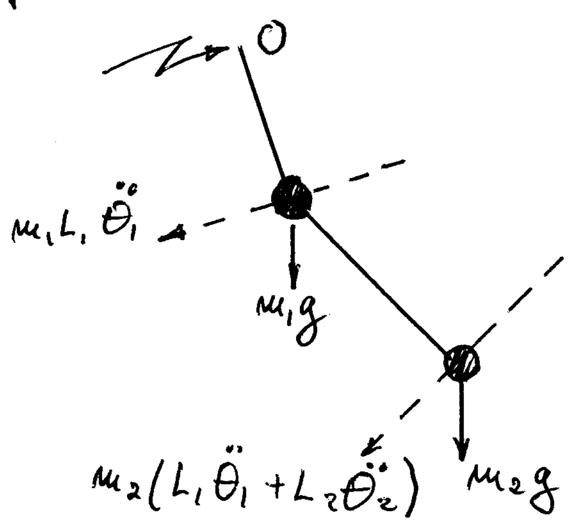


Diagram benda bebas massa m_2:

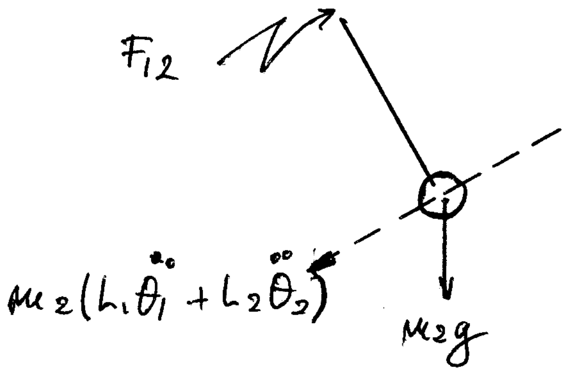
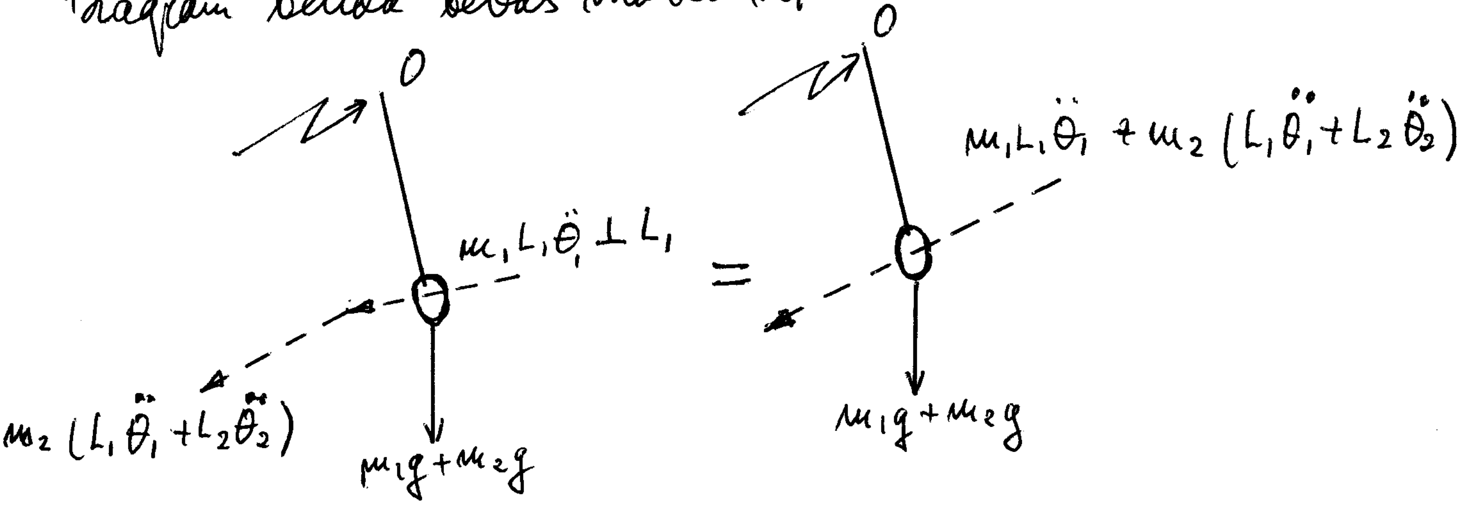


Diagram benda bebas massa m_1:



Persamaan gerak m_1:

$$[m_1 L_1 \ddot{\theta}_1 + m_2(L_1\ddot{\theta}_1 + L_2\ddot{\theta}_2)]L_1 + (m_1 g + m_2 g)L_1\theta_1 = 0$$

$$m_2 L_2(L_1\ddot{\theta}_1 + L_2\ddot{\theta}_2) + m_2 g L_2 \theta_2 = 0$$

$$m_1 L_1^2 \ddot{\theta}_1 + m_2 L_1^2 \ddot{\theta}_1 + \underline{m_2} L_1 L_2 \ddot{\theta}_2 + (m_1 g L_1 + m_2 g L_1)\theta_1 = 0$$

Persamaan gerak m_2 (momen thd m_1 dari m_2):

$$m_2 L_1 L_2 \ddot{\theta}_1 + m_2 L_2^2 \underline{\ddot{\theta}_2} + m_2 g L_2 \theta_2 = 0.$$

Dalam bentuk matriks

$$\begin{bmatrix} m_1 L_1^2 + m_2 L_1^2 & m_2 L_1 L_2 \\ m_2 L_1 L_2 & m_2 L_2^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} m_1 g L_1 + m_2 g L_1 & 0 \\ 0 & m_2 g L_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = 0 \quad \cdots (1)$$

Dengan mengsubstitusikan $\ddot{\theta}_i = -\omega^2 \Theta_i$, maka

$$m_1 L_1^2 (-\omega^2 \Theta_1) + m_2 L_1^2 (-\omega^2 \Theta_1) + m_2 L_1 L_2 (-\omega^2 \Theta_2) + (m_1 g L_1 + m_2 g L_2)\Theta_1 = 0$$

$$m_2 L_1 L_2 (-\omega^2 \Theta_1) + m_2 L_2^2 (-\omega^2 \Theta_2) + m g L_2 \Theta_2 = 0$$

Bentuk matriks

$$\begin{bmatrix} m_1 g L_1 + m_2 g L_2 - \omega^2 (m_1 L_1^2 + m_2 L_1^2) & -\omega^2 (m_2 L_1 L_2) \\ -\omega^2 m_2 L_1 L_2 & -\omega^2 m_2 L_2^2 + m_2 g L_2 \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Persamaan frekuensi sistem:

$$\Delta(\omega) = \begin{vmatrix} m_1 g L_1 + m_2 g L_2 - \omega^2 (m_1 L_1^2 + m_2 L_1^2 & -\omega^2 (m_2 L_1 L_2) \\ -\omega^2 m_2 L_1 L_2 & -\omega^2 m_2 L_2^2 + m_2 g L_2 \end{vmatrix} = 0$$

Jika diselsaikan akan diperoleh:

$$\omega^4 m_1 L_1 L_2 - \omega^2 (m_1 g L_2 + m_2 g L_2 + m_1 L_1 g + m_2 L_1 g) + m_1 g^2 + m_2 g^2 = 0$$

$A = m_1 L_1 L_2$

$B = (m_1 g L_2 + m_2 g L_2 + m_1 L_1 g + m_2 L_1 g)$

$C = m_1 g^2 + m_2 g^2$

$$\omega_{1.2}^2 = g \frac{m_1 L_2 + m_2 L_2 + m_1 L_1 + m_2 L_1 \pm \sqrt{B^2 - 4AC}}{2 m_1 L_1 L_2}$$

Jika $m_2 = 2m$; $m_1 = m$; $L_1 = L_2 = L$; $g = 10$, maka

Analog:

$$\omega_{1.2}^2 = g \frac{3 \pm \sqrt{6}}{L}$$

$$\omega_1^2 = 0{,}550 \frac{g}{L} \; ; \; \omega_2^2 = 5{,}4495 \frac{g}{L}$$

Dari matriks (1):

$$\begin{bmatrix} 3mL^2 & 2mL^2 \\ 2mL^2 & 2mL^2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 3mgL & 0 \\ 0 & 2mgL \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = 0 \rightarrow \text{dibagi}(L)$$

$$\begin{bmatrix} 3mL & 2mL \\ 2mL & 2mL \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 3mg & 0 \\ 0 & 2mg \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = 0$$

Subtitusi dengan $\ddot{\theta}_i = -W^2 \Theta_i$ maka

$$-3mLW^2\Theta_1 - 2mLW^2\Theta_2 + 3mg\Theta_1 = 0$$

$$\frac{\Theta_1}{\Theta_2} = \frac{-2LW^2}{3g - 3LW^2}$$

Untuk $W_1^2 = 0{,}5505\ g/L$, maka

$$\frac{\Theta_1}{\Theta_2} = \frac{-2 \cdot \cancel{L} \cdot \frac{0{,}5505\ g}{\cancel{L}}}{3g - 3\cancel{L} \cdot \frac{0{,}5505}{\cancel{L}} g} = \frac{-2 \cdot 0{,}5505 \cdot g}{3g - 3 \cdot 0{,}5505 g} =$$

$$= \frac{-1{,}101\ g}{1{,}3485\ g} = -0{,}816$$

(2) Maka modus getar pertama:

$$[U]_1 = \begin{bmatrix} 1 \\ -0{,}816 \end{bmatrix}$$

Untuk $W_2^2 = 5{,}4495 \frac{g}{L}$

$$\frac{\Theta_1}{\Theta_2} = \frac{-2\cancel{L} \cdot 5{,}4495 \frac{g}{\cancel{L}}}{3g - 3\cancel{L} \frac{5{,}4495\ g}{\cancel{L}}} = \frac{-10{,}899\ g}{3g - 3 \cdot 5{,}4495 \cdot g} = 0{,}8165$$

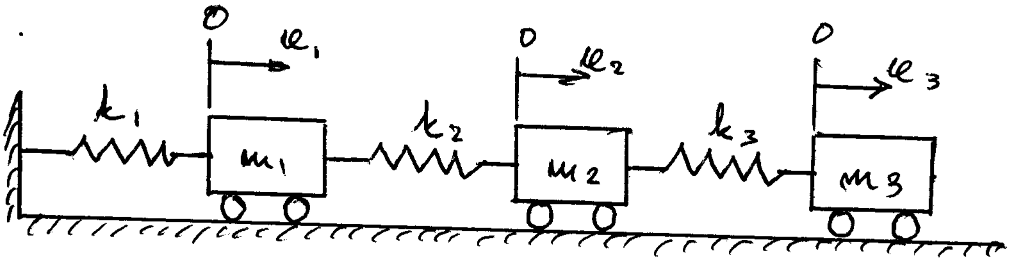
Modus getar kedua:

$$[U]_2 = \begin{bmatrix} 1 \\ 0{,}8165 \end{bmatrix}$$

Matriks modus getar

$$[U] = \begin{bmatrix} 1 & 1 \\ -0{,}816 & 0{,}8165 \end{bmatrix}$$

Tentukan persamaan gerak, modus getar dan frekuensi natural untuk sistem pegas massa 3 derajat kebebasan spt gambar dibawah ini.



Jawab:

Persamaan gerak untuk massa 1:

$$m_1\ddot{u}_1 + k_1 u_1 - k_2(u_2 - u_1) = 0$$

$$m_1\ddot{u}_1 + (k_1+k_2)u_1 - k_2 u_2 = 0 \quad \dots \text{(1)}$$

Persamaan gerak untuk massa 2:

$$m_2\ddot{u}_2 + k_2(u_2-u_1) - k_3(u_3-u_2) = 0$$

$$m_2\ddot{u}_2 - k_2 u_1 + (k_2+k_3)u_2 - k_3 u_3 = 0 \quad \dots \text{(2)}$$

Persamaan gerak untuk massa 3:

$$m_3\ddot{u}_3 + k_3(u_3-u_2) - \cancel{k_4(u_4-u_3)} = 0$$

$$m_3\ddot{u}_3 - k_3 u_2 + k_3 u_3 = 0 \quad \dots \text{(3)}$$

Dalam bentuk matriks:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \dots \text{(4)}$$

Jika $m_1 = m_2 = m_3 = m$ dan $k_1 = k_3 = k$; $k_2 = 2k$, maka:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix} + \begin{bmatrix} 3k & -2k & 0 \\ -2k & 3k & -k \\ 0 & -k & k \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Subtitusi dengan $u_i = X_i e^{J\omega t}$ dan $\ddot{u}_i = -\omega^2 X_i e^{J\omega t}$

$$\begin{bmatrix} 3k - m\omega^2 & -2k & 0 \\ -2k & 3k - m\omega^2 & -k \\ 0 & -k & k - m\omega^2 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Jika diselesaikan akan didapat persamaan frekuensi:

$$\omega^6 - 7\left(\frac{k}{m}\right)\omega^4 + 10\left(\frac{k}{m}\right)^2\omega^2 - 2\left(\frac{k}{m}\right)^3 = 0.$$

Dengan menggunakan MATLAB:

```
roots ([1  -7  10  -2])
ans =  5,1249 = ω3²
       1,6367 = ω2²
       0,2384 = ω1²
```

Modus getar didapat dg membandingkan X_1 & X_2 serta X_2 & X_3 untuk masing² frekuensi:

Frekuensi: $\omega_1^2 = 0{,}2384\left(\frac{k}{m}\right)$

Gunakan ① untuk membandingkan X_1 & X_2 dan substitusi $x_i = X_i e^{j\omega t}$ dan $\ddot{x}_i = -\omega^2 X_i e^{j\omega t}$, maka:

$$(k_1 + k_2 - m\omega^2)X_1 - k_2 X_2 = 0$$

$$(3k - m\omega^2)X_1 - 2kX_2 = 0$$

$$\frac{X_1}{X_2} = \frac{2k}{3k - m\omega^2} = \frac{2k}{3k - 0{,}2384k} = 0{,}7242$$

Dengan menggunakan ③:

$$m_3\ddot{x}_3 - k_3x_2 + k_3x_3 = 0$$

$$-m\omega^2X_3 - kX_2 + kX_3 = 0$$

$$\frac{X_2}{X_3} = \frac{k - m\omega^2}{k} = \frac{k - 0{,}2384k}{k} = 0{,}7616$$

Modus getar pertama:

$$u_1 = \begin{bmatrix} 1 \\ 1/0{,}7242 \\ 1/(0{,}7242 \cdot 0{,}7616) \end{bmatrix} = \begin{bmatrix} 1 \\ 1{,}3808 \\ 1{,}8131 \end{bmatrix}$$

Frekuensi $\omega_2^2 = 1,6367 \left(\frac{k}{m}\right)$

$$\frac{X_1}{X_2} = \frac{2k}{3k - m\omega^2} = \frac{2k}{3k - 1,6367k} = 1,467$$

$$\frac{X_2}{X_3} = \frac{k - m\omega^2}{k} = \frac{k - 1,6367k}{k} = -0,6367$$

Modus getar kedua:

$$U_2 = \begin{bmatrix} 1 \\ 1/1,467 \\ 1/(1,467 \cdot (-0,6367)) \end{bmatrix} = \begin{bmatrix} 1 \\ 0,6817 \\ -1,0706 \end{bmatrix}$$

Frekuensi $\omega_3^2 = 5,1249 \left(\frac{k}{m}\right)$

$$\frac{X_1}{X_2} = \frac{2k}{3k - m\omega^2} = \frac{2k}{3k - 5,1249k} = -0,9412$$

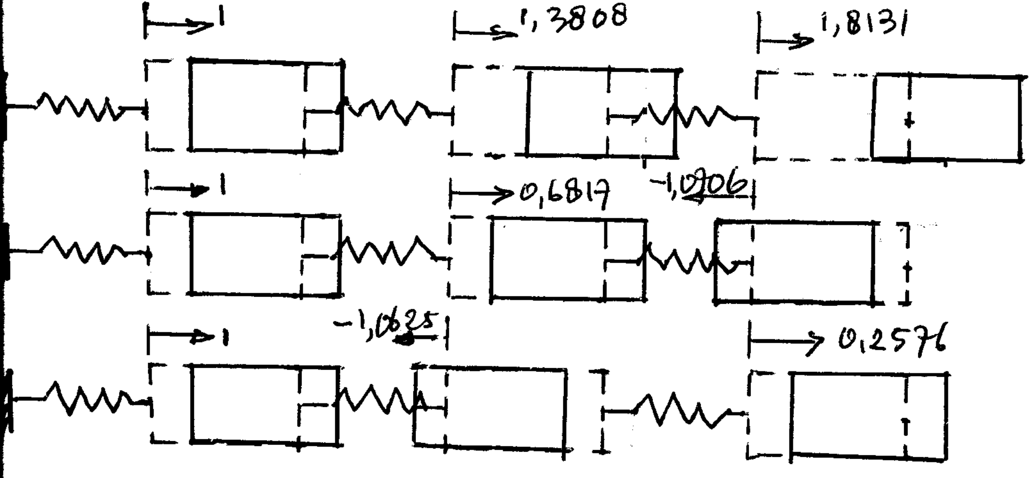
$$\frac{X_2}{X_3} = \frac{k - m\omega^2}{k} = \frac{k - 5,1249k}{k} = -4,1249$$

Modus getar ketiga

$$U_3 = \begin{bmatrix} 1 \\ 1/-0,9412 \\ 1/(-0,9412 \cdot -4,1249) \end{bmatrix} = \begin{bmatrix} 1 \\ -1,0625 \\ 0,2576 \end{bmatrix}$$

Maka vektor modus getar:

$$U = \begin{bmatrix} 1 & 1 & 1 \\ 1,3808 & 0,6817 & -1,0625 \\ 1,8131 & -1,0706 & 0,2576 \end{bmatrix}$$



modus ke 1

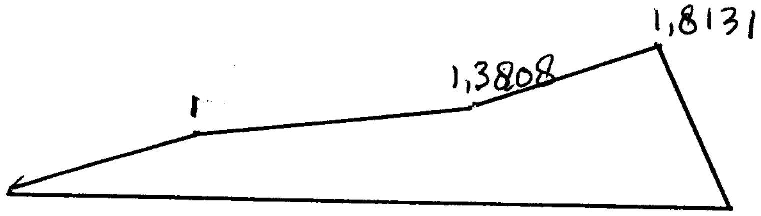
modus ke 2

modus ke 3

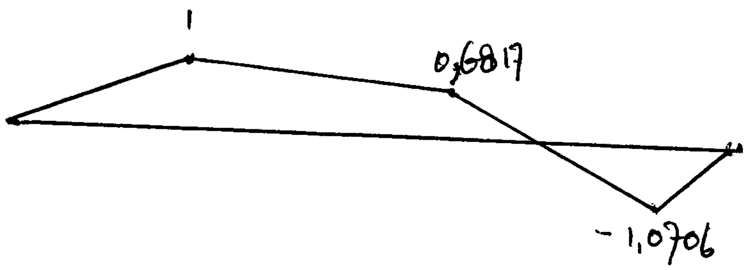
Dengan menggunakan prinsip superposisi modus, gerak sistem adalah:

$$x = \begin{bmatrix} 1 \\ 1,3808 \\ 1,8131 \end{bmatrix} X_{11} \sin(\omega_1 t + \psi_1) + \begin{bmatrix} 1 \\ 0,6817 \\ -1,0706 \end{bmatrix} X_{12} \sin(\omega_2 t + \psi_2) + \begin{bmatrix} 1 \\ -1,0625 \\ 0,2576 \end{bmatrix} X_{13} \sin(\omega_3 t + \psi_3)$$

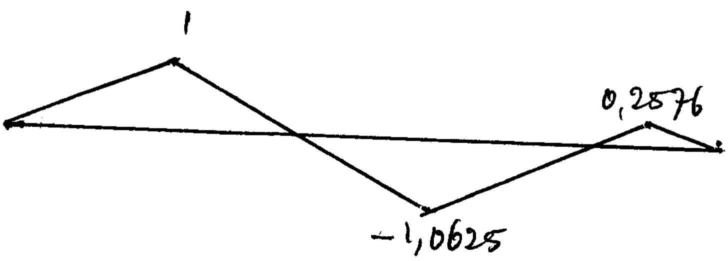
dimana X_{11}; X_{12}; X_{13} dan ψ_1; ψ_2; ψ_3 diperoleh dari kondisi awal:



modus ke 1

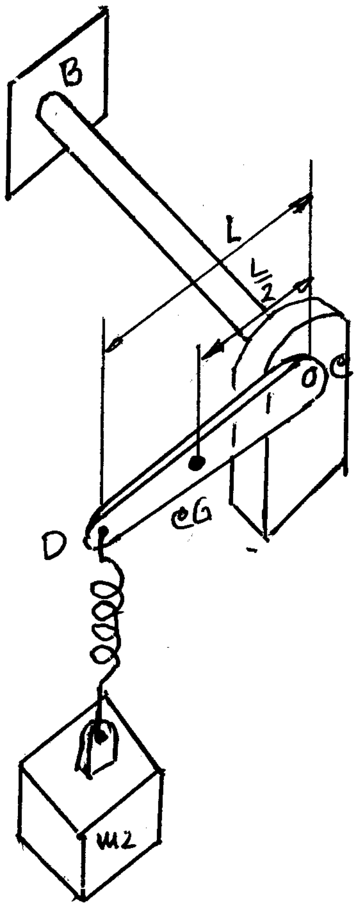


modus ke 2



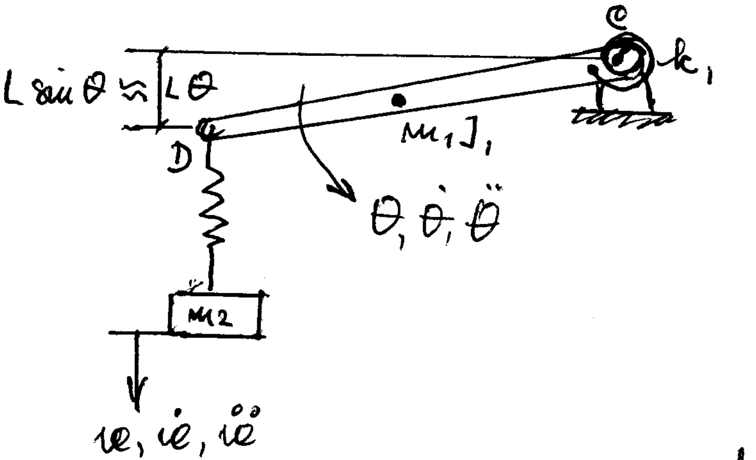
modus ke 3

1) Tentukan persamaan getar sist pd gambar dibawah ini. Batang CD dianggap benda tegar dg massa m_1 dan momen inersia massa J_1.

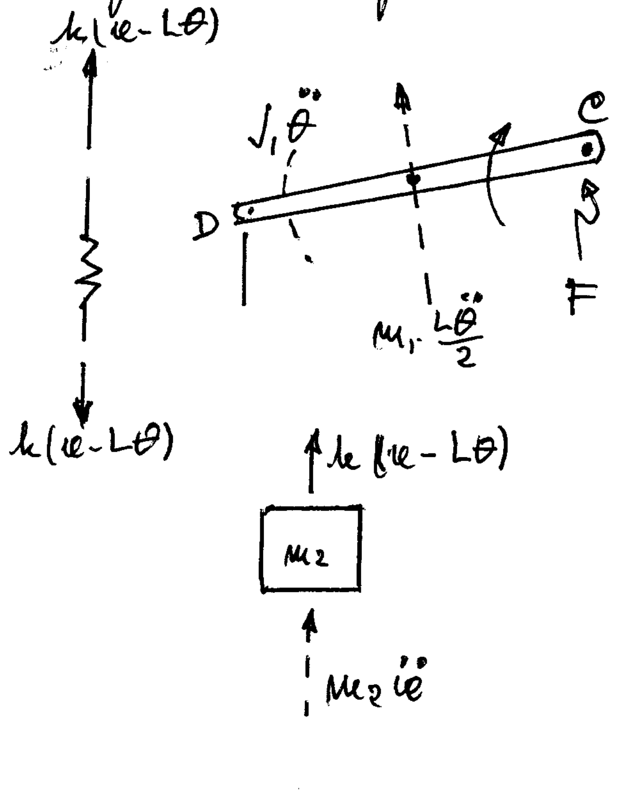


Jawab.

Dibuat sederhana :



Gambar diagram benda bebas:



Persamaan gerak untuk batang CD:

$$\Sigma M_C = 0$$

$$J_1 \ddot{\theta} + m\ddot{\theta}\cdot\frac{L}{2}\cdot\frac{L}{2} + k_1\theta - k(x - L\theta)L = 0$$

$$\left[J_1 + m_1\left(\frac{L}{2}\right)^2\right]\ddot{\theta} + k_1\theta - k(x - L\theta)L = 0$$

Misal $J_1 + m_1\left(\frac{L}{2}\right)^2 = J_0$, maka:

$$J_0 \cdot \ddot{\theta} + k_1\theta - kL(x - L\theta) = 0$$

Persamaan gerak untuk m_2:

$$m_2 \cdot \ddot{x} + k(x - L\theta) = 0.$$

Dalam bentuk matriks pers. gerak sistem:

$$\begin{bmatrix} J_0 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} k_1 + kL^2 & -kL \\ -kL & -k \end{bmatrix}\begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Subtitusi $\ddot{\theta} = -W^2 \cdot \theta$ didapat

$$-J_0 W^2 \cdot \theta\,(k + kL^2)\theta - kL\,x = 0$$

$$(k + kL^2 - J_0 W^2)\theta = kL \cdot x \quad \ldots \text{(1)}$$

Persamaan frekuensi

$$D(W) = \begin{bmatrix} k_1 + kL^2 - J_0 W^2 & -kL \\ -kL & k - m_2 W^2 \end{bmatrix} = 0.$$

$$J_0 m_2 W^4 - (m_2 k_1 + m_2 kL^2 + J_0 k)W^2 + k k_1 = 0$$

$$A = J_0 m_2$$

$$B = -(m_2 k_1 + m_2 kL^2 + J_0 k)$$

$$C = k \cdot k_1.$$

$$W_{1\text{-}2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{m_2 k_1 + m_2 kL^2 + J_0 k \pm \sqrt{(m_2 k_1 + m_2 kL^2 + J_0 k)^2 - 4 J_0 m_2 k}}{}$$

$$= \frac{m_2 k_1 + m_2 kL^2 + J_0 k \pm \sqrt{(m_2 k_1 + m_2 kL^2 + J_0 k)^2 - 4 J_0 m_2 k k_1}}{2 J_0 m_2}$$

Dari (1) didapat perbandingan responnya:

$$\frac{\theta}{x} = \frac{kL}{k_1 + kL^2 - J_0 W^2}.$$

Berikutnya adalah gunakan cara yg sama.