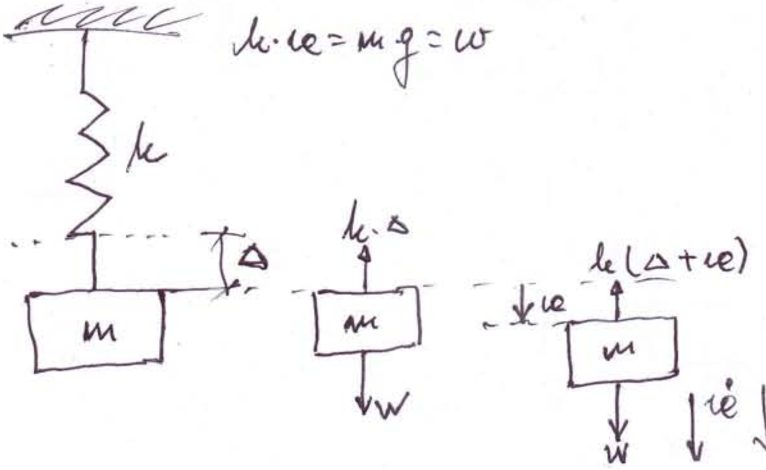
Getaran.

① Getaran bebas tak teredam (tanpa gesekan udara) SDOF:
Massa pegas diabaikan.



$$\text{Periode} = 2\pi\sqrt{\frac{m}{k}}$$

$$\omega_n = 2\pi f \longrightarrow f = \frac{\omega_n}{2\pi}$$

$$\omega_n = \sqrt{\frac{k}{m}} \;\ldots \text{rad/s}$$

$$\sqrt{\frac{k}{m}} = 2\pi f$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \;\ldots \text{Hz}$$

Dengan metode energi:

$$KE = \frac{1}{2} m \cdot \dot{x}^2 \;;\; PE = \frac{1}{2}mgh = \frac{1}{2}kx \cdot x = \frac{1}{2}kx^2$$

bergerak sejauh x

$$KE + PE = C$$

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$$

$$\frac{d}{dt}(KE + KP) = 0$$

$$m\dot{x}\cdot\ddot{x} + kx\dot{x} = 0$$

$\therefore$ Pers. gerak: $m\cdot\ddot{x} + kx = 0$

$$PE = \int_0^x (\text{total spring force})\,dx - mgx$$

$$= \int_0^x (mg + kx)\,dx - mgx$$

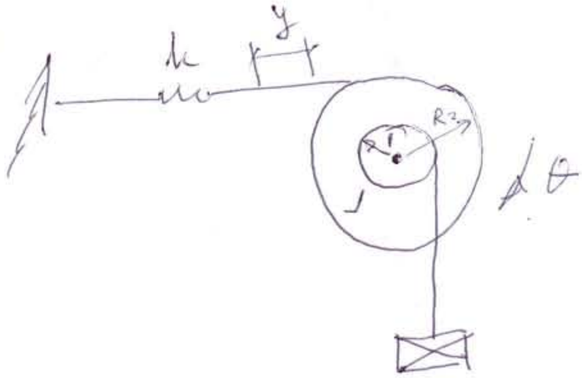
$$= \left. mgx + \frac{1}{2}kx^2 \right]_0^x - mgx$$

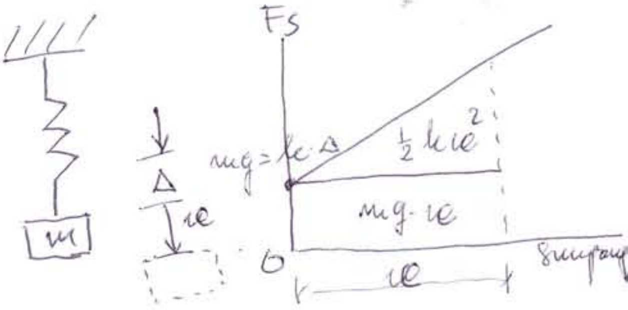
$$= \frac{1}{2}kx^2$$

Metode Energi

$T + U = C \rightarrow T$: energi kinetik $= \frac{1}{2} m$...
U : energi potensial $= mg$...

$\frac{d}{dt}(T+U) = 0$.

② 

① 

① Karena simpangan x, energi potensial pegas = luasan gambar diatas $mgx + \frac{1}{2} k \cdot x^2$.

Hilangnya energi potensial dari m karena simpangan x yaitu $-mgx$, shg perubahan neto energi potensial menjadi $\frac{1}{2} kx^2$

$T + U = C$

$\frac{1}{2} m \cdot \dot{x}^2 + \frac{1}{2} kx^2 = C$, diturunkan menjadi

$m \cdot \ddot{x} + kx = 0$

② $U = \frac{1}{2} kx^2$ $\quad T_{maks} = \frac{1}{2} m \cdot \dot{x}^2 + \frac{1}{2} \cdot J\dot{\theta}^2 = \frac{1}{2} m \cdot (r_1\dot{\theta})^2 + \frac{1}{2} J\dot{\theta}^2$

$r_1\theta = x$ $\quad U_{maks} = \frac{1}{2} K y^2 = \frac{1}{2} K (r_2\theta)^2$

$r_1\dot{\theta} = \dot{x}$ $\quad T + U = \frac{1}{2} m(r_1\dot{\theta})^2 + \frac{1}{2} J\dot{\theta}^2 + \frac{1}{2} K(r_2\theta)^2$

$r_2\theta = y$ $\quad \frac{d}{dt}(T+U) = \frac{1}{2} m \cdot r_1^2 \cdot 2 \cdot \dot{\theta} \frac{d\dot{\theta}}{dt} + \frac{1}{2} J \cdot 2 \cdot \dot{\theta} \frac{d\dot{\theta}}{dt} + \frac{1}{2} K \cdot r_2^2 \cdot 2\theta$...

$r_2\dot{\theta} = \dot{y}$

$= m r_1^2 \cdot \dot{\theta} \cdot \ddot{\theta} + J\dot{\theta}\ddot{\theta} + k r_2^2 \theta \cdot \dot{\theta} = 0$

$= \dot{\theta} \{(m r_1^2 + J)\ddot{\theta} + k r_2^2 \cdot \theta\} = 0$

untuk $\dot{\theta} \neq 0 \rightarrow$ maka

$(mr_1^2 + J)\ddot{\theta} + kr_2^2 \cdot \theta = 0$ identik dg (2)

$\therefore \ddot{x} + \omega_n^2 x = 0$

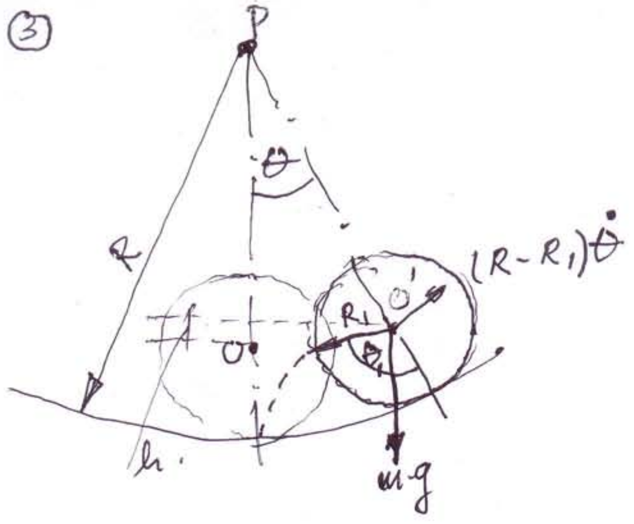
$\omega_n^2 = \frac{k}{m}$

$m\ddot{x} + kx = 0 \quad : m$

$\ddot{x} + \frac{k}{m} \cdot x = 0$

$\ddot{x} + \omega_n^2 x = 0 \longrightarrow$ maka

$$\boxed{\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kr_2^2}{mr_1^2 + J}}}$$



Gerak rotasi pusat massa:

$x = x_0 \sin\theta = O'O \sin\theta$.

Kecepatan translasi bagi pusat massa sil

$\dot{x} = O'O \cos\theta\,\dot{\theta}$

$\dot{x}_{max}$ pada saat $\theta = 0^\circ = 1$

$\dot{x}_{max} = O'O.\dot{\theta} = (R-R_1)\dot{\theta} = V.$

Kecepatan sudut rotasi silinder $= \dot{\theta}_1 - \dot{\theta} =$

Karena silinder menggelinding tanpa slip:

$$R\theta = R_1\theta_1$$

$$\theta_1 = \frac{R}{R_1}.\theta$$

$$\dot{\theta}_1 = \frac{R}{R_1}.\dot{\theta}$$

Kecepatan sudut:

$$\dot{\theta}_1 - \dot{\theta} = \left[\left(\frac{R}{R_1}.\dot{\theta}\right) - \dot{\theta}\right] = \left(\frac{R}{R_1} - 1\right)\dot{\theta}$$

Energi kinetik (T):

$$T = \frac{1}{2}mV^2 + \frac{1}{2}J(\dot{\theta}_1 - \dot{\theta})^2 \rightarrow J = \text{inersia} = \frac{1}{2}mR_1^2.$$

$$= \frac{1}{2}m\left[(R-R_1)\dot{\theta}\right]^2 + \frac{1}{2}.\frac{1}{2}mR_1^2(\dot{\theta}_1 - \dot{\theta})^2$$

$$= \frac{1}{2}m\left[(R-R_1)\dot{\theta}\right]^2 + \frac{1}{4}m.R_1^2.(\dot{\theta}_1 - \dot{\theta})^2$$

$$= \frac{1}{2}m\left[(R-R_1)\dot{\theta}\right]^2 + \frac{1}{4}mR_1^2.\left[\left(\frac{R}{R_1} - 1\right)\dot{\theta}\right]^2$$

$$= \frac{1}{2}m\left[(R-R_1)\dot{\theta}\right]^2 + \frac{1}{4}m.\frac{\cancel{R_1^2}}{\cancel{R_1^2}}\left[(R-R_1)^2.\dot{\theta}^2\right]$$

$$= \frac{1}{2}m\left[(R-R_1)^2.\dot{\theta}^2\right] + \frac{1}{4}m\left[(R-R_1)^2.\dot{\theta}^2\right]$$

$$= \frac{3}{4}m\left[(R-R_1)^2.\dot{\theta}^2\right]$$

$R_1^2\left(\frac{R^2}{R_1^2} - 1^2\right) = \frac{R_1^2}{R_1^2}(R^2 - R_1^2)$

$= (R-R_1)^2$

ok

$R_1^2\left(\frac{R^2}{R_1^2} - \frac{R_1^2}{R_1^2}\right)$

$R_1^2\left(\frac{R^2 - R_1^2}{R_1^2}\right)$

$\frac{R_1^2}{R_1^2}(R^2 - R_1^2)^2$

Energi Potensial (U): sebelum bergerak $h = [(R-R_1) - (R-R_1)\cos\theta]$

$$U = mgh.$$

$$= m.g.\left[(R-R_1) - (R-R_1)\cos\theta\right]$$

$$= m.g.\left[(R-R_1)(1-\cos\theta)\right]$$

$$T+U = \frac{3}{4} m \left[(R-R_1)^2 \cdot \dot{\theta}^2 \right] + mg\left[(R-R_1)(1-\cos\theta) \right] = 0$$

$$\frac{d}{dt}(T+U) = \frac{3}{4} m \left[(R-R_1)^2 \cdot 2\dot{\theta} \right] \frac{d\dot{\theta}}{dt} + mg\left[(R-R_1) \sin\theta \cdot \frac{d\theta}{dt} \right] = 0$$

$$= \frac{3}{4} m \left[(R-R_1)^2 \cdot 2\cancel{\dot{\theta}} \right] \ddot{\theta} + mg\left[(R-R_1) \sin\theta \cdot \cancel{\dot{\theta}} \right] = 0$$

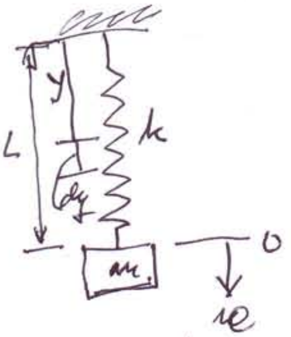
$\sin\theta = \theta \rightarrow \dot{\theta} \neq$ (küçük)

$$= \underbrace{\frac{3}{2} m \left[(R-R_1)^2 \right]}_{m} \ddot{\theta} + \underbrace{m \cdot g \left[(R-R_1) \right]}_{k} \cdot \theta = 0$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{\cancel{m} \cdot g \left[\cancel{(R-R_1)} \right]}{\frac{3}{2} \cancel{m} \left[(R-R_1)^{\cancel{2}} \right]}} = \boxed{\sqrt{\frac{2g}{3(R-R_1)}}}$$

(4) (5)

Metode RAYLEIGH:



$\dot{u}$ = kecepatan massa m = kecepat pegas dari jarak y.

$\dot{u}\frac{y}{L}$ ... (1)

$$T = \frac{1}{2}\int_0^L \left(\dot{u}\frac{y}{L}\right)^2 \frac{m_s}{L}\,dy$$

$$= \frac{1}{2}\dot{u}^2 \cdot \frac{1}{3}y^3 \cdot L^{-2} \cdot L^{-1} \cdot m_s\Big]_0^L$$

$$= \frac{1}{2}\dot{u}^2 \cdot \frac{1}{3}\cancel{L^3} \cdot \cancel{L^{-3}}\, m_s \quad \text{ternyata:}$$

$$= \frac{1}{2}\left(\frac{m_s}{3}\right)\dot{u}^2 \rightarrow m_{ef} = \frac{1}{3}m_s$$

m_{ef} = massa efektif
m_s = massa pegas

$$\omega_n = \sqrt{\frac{k}{m + \frac{1}{3}m_s}}$$

Displasemen ujung pegas = $u(t)$, maka displasemen:

$$y = \frac{y}{L}u(t)$$

Ekinetik = EK pegas + EK massa

$$EK \text{ pegas } * dy = \frac{1}{2}\rho\, dy\left[\frac{y}{L}\dot{u}(t)\right]$$

$\rho = \frac{\text{massa}}{\text{panjang}}$

$u = A \sin \omega_n t$

$$T = \frac{1}{2}m\,\dot{u}_{max}^2 + \int_0^L \frac{1}{2}\rho\left(\frac{y}{L}\dot{u}_{max}\right)^2 dy$$

$$= \frac{1}{2}m\,\dot{u}_{max}^2 + \frac{1}{2}\rho\,\frac{1}{3}y^3 \cdot L^{-2}\,\dot{u}_{max}^2\Big]_0^L$$

$$= \frac{1}{2}m \cdot \dot{u}_{max}^2 + \frac{1}{2}\rho \cdot \frac{1}{3}L^3 \cdot L^{-2}\,\dot{u}_{max}^2$$

$$= \frac{1}{2}m \cdot \dot{u}_{max}^2 + \frac{1}{2}\rho \cdot \frac{L}{3} \cdot \dot{u}_{max}^2.$$

$$= \frac{1}{2}\left(m + \frac{\rho L}{3}\right)\dot{u}_{max}^2.$$

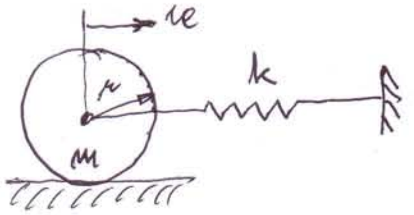
$$= \frac{1}{2}\left(m + \frac{\rho L}{3}\right)(\omega_n \cdot A)^2.$$

$$U = \frac{1}{2}k\,u_{max}^2 = \frac{1}{2}k \cdot A^2$$

Rayleigh method:

$$\frac{1}{2}\left(m + \frac{\rho L}{3}\right)(\omega_n \cdot A)^2 = \frac{1}{2}k \cdot A^2$$

$$\therefore \omega_n = \sqrt{\frac{\cancel{\frac{1}{2}}k \cdot \cancel{A^2}}{\cancel{\frac{1}{2}}\left(m + \frac{\rho L}{3}\right)(\cancel{A^2})}} = \sqrt{\frac{k}{\left(m + \frac{\rho L}{3}\right)}}$$



Energi kinetik translasi $= \frac{1}{2} m \cdot \dot{x}^2$

Energi kinetik rotasi $= \frac{1}{2} J_0 \cdot \dot{\theta}^2$

J_0 = momen inersia silinder $= \frac{1}{2} m r^2$.

$x = r\theta$

$\dot{x} = r\dot{\theta}$

$\dot{\theta} = \frac{\dot{x}}{r}$

$$K.E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} \cdot \frac{1}{2} m r^2 \cdot \left(\frac{\dot{x}}{r}\right)^2$$
$$= \frac{3}{4} m \dot{x}^2$$

$$PE = \frac{1}{2} k \cdot x^2$$

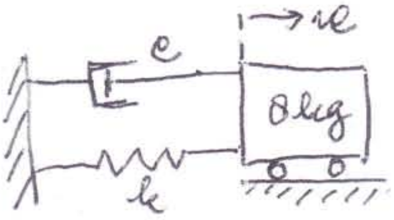
$$\frac{d}{dt}(KE + PE) = 0$$
$$\frac{d}{dt}\left(\frac{3}{4} m \cdot \dot{x}^2 + \frac{1}{2} k \cdot x^2\right) = 0$$
$$2 \cdot \frac{3}{4} m \cdot \dot{x} \cdot \ddot{x} + 2 \cdot \frac{1}{2} k \cdot x \cdot \dot{x} = 0$$
$$\left(\frac{3}{2} m \cdot \ddot{x} + kx\right)\dot{x} = 0 \rightarrow (3m \cdot \ddot{x} + 2kx)\dot{x} = 0$$
$$\omega_n = \sqrt{\frac{2k}{3m}} \text{ rad/sec.}$$

Cara Newton II :

F_f = gaya gesek

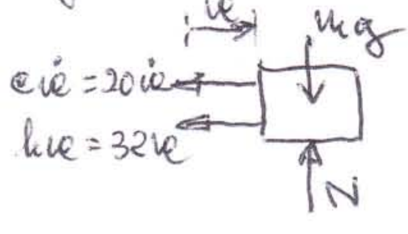
$$\sum F = m a$$
$$= m \cdot \ddot{x} = -kx + F_f$$

$$F_f = -\frac{1}{2} m \ddot{x}$$
$$\therefore m \ddot{x} = -kx - \frac{1}{2} m \ddot{x}$$
$$\frac{3}{2} m \ddot{x} + kx = 0.$$
$$\omega_n = \sqrt{\frac{2k}{3m}} \text{ rad/s}$$



Benda dengan masa 8 kg dipindah kekanan 0,2 m dan dilepas pd waktu t = 0
Tentukan perpindahan t = 2"
Koefisien redaman c = 20 N det/m
Kekakuan pegas k = 32 N/m.

Jawab



$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{32}{8}} = 2 \text{ rad/s}$$

$$\xi = \frac{c}{2m\omega_n} = \frac{20}{2 \cdot 8 \cdot 2} = 0,625$$

Karena $\xi < 1$, maka termasuk getaran dg redaman subkritis

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 2 \cdot \sqrt{1-0,625^2} = 1,561 \text{ rad/s}$$

Persamaan gerakan :

$$x = e^{-\xi \omega_n t}(B_1 \cos \omega_d t + B_2 \sin \omega_d \cdot t)$$

Kondisi awal $x = 0,2 \text{ m} \rightarrow t = 0$

$$x = e^{-0,625 \cdot 2 \cdot 0}(B_1 \cdot \cos 0 + B_2 \sin 0)$$

$$x = B_1 + 0$$

$$\underline{\underline{0,2 = B_1}}$$

Persamaan kecepatan :

$$\dot{x} = -\xi \omega_n \cdot e^{-\xi \omega_n t}(B_1 \cos \omega_d t + B_2 \sin \omega_d \cdot t)$$
$$+ e^{-\xi \omega t}(-\omega_d \cdot B_1 \sin \omega_d t + \omega_d B_2 \cdot \cos \omega_d t)$$

$$0 = -\xi \omega_n \cdot e^0 (0,2 \cos 0 + B_2 \cdot \sin 0)$$
$$+ e^0(-\omega_d \cdot B_1 \sin 0 + \omega_d \cdot B_2 \cdot \cos 0)$$

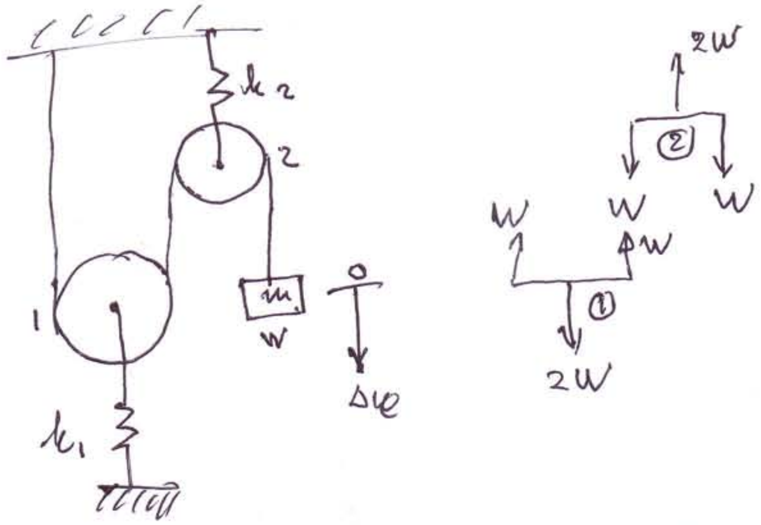
$$0 = (-0,2 \cdot \xi \omega_n + 0) + (0 + \omega_d \cdot B_2 \cdot 1)$$

$$0 = -0,2 \cdot \xi \omega_n + 1,561 \cdot B_2$$

$$B_2 = \frac{0,2 \cdot 0,625 \cdot 2}{1,561} = \underline{\underline{0,1602}}$$

Jadi $x_{t=2} = e^{-\xi \omega_n t}(0,2 \cos \overbrace{1,561 \cdot t}^{\text{radial}} + 0,1602 \sin \overbrace{1,561 \cdot t}^{\text{radial}}) = \underline{\underline{0,0162 \text{ m}}}$

7



Titik pulley 1 bergerak sejauh $2W/k_1$

— " — 2 — " — " — $2W/k_2$

Total perpindahan massa m :

$$2\left(\frac{2W}{k_1}+\frac{2W}{k_2}\right)$$

k equivalen pegas :

$$\frac{W}{k_{eq}} = 4W\left(\frac{1}{k_1}+\frac{1}{k_2}\right) = \frac{4W(k_1+k_2)}{k_1 k_2}$$

$$\frac{1}{k_{eq}} = \frac{4(k_1+k_2)}{k_1 \cdot k_2}$$

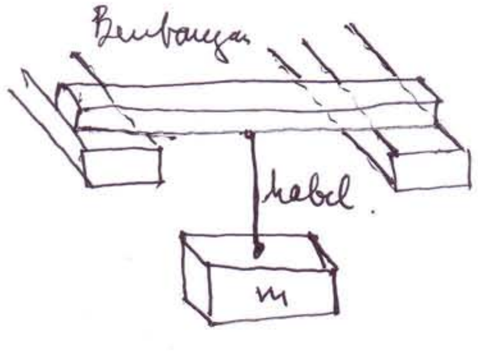
$$k_{eq} = \frac{k_1 \cdot k_2}{4(k_1+k_2)}$$

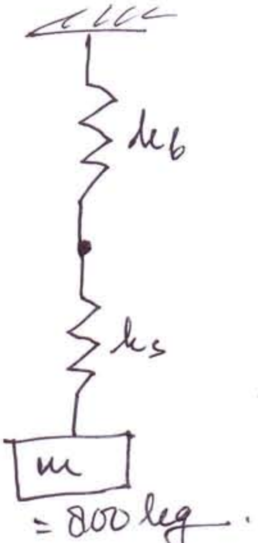
Persamaan getaran dg kekakuan equivalen :

$$m\ddot{x} + k_{eq}x = 0$$

$$\therefore \omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_1 \cdot k_2}{4m(k_1+k_2)}} \ \text{rad/s} \quad \text{atau}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1 \cdot k_2}{4m(k_1+k_2)}} \ \text{Hz}.$$





$L = 3{,}1\ \text{m}$

Bentangan: $E = 200 \times 10^{9}\ \text{N/m}^2$

$I = 3{,}5 \times 10^{-4}\ \text{m}^4$

Kabel: $E = 200 \times 10^{7}\ \text{N/m}^2$

$r = 10\ \text{cm}.$

Panjang kabel 9 m.

Kekakuan Bentangan: $k_b = \dfrac{48EI}{L^3} = \dfrac{48(200\times10^{9})(3{,}5\times10^{-4})}{(3{,}1)^3} = 1{,}13\cdot10^{8}\ \text{N/m}$

Kekakuan kabel $k_s = \dfrac{A\cdot E}{L} = \dfrac{\pi(0{,}1)^2(200\times10^{7})}{9} = 6{,}98\times10^{8}\ \text{N/m}.$

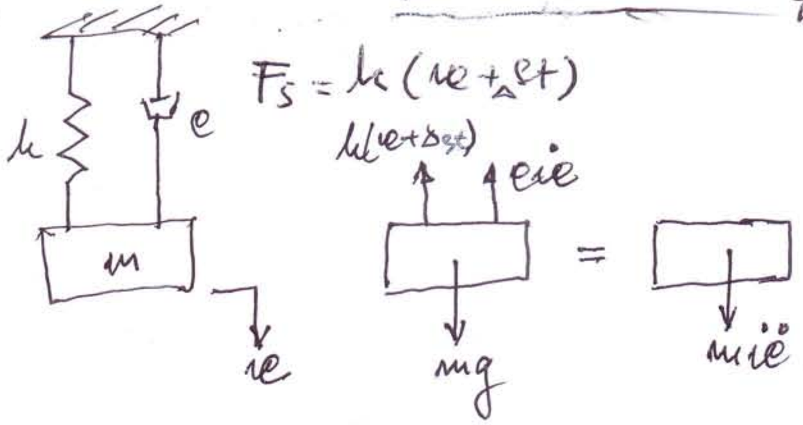
Kekakuan bentangan dan kabel dipasang seri:

$$k_{eq} = \frac{1}{\frac{1}{k_b}+\frac{1}{k_s}} = \frac{1}{\frac{1}{1{,}13\times10^{8}}+\frac{1}{6{,}98\times10^{8}}} = 9{,}73\cdot10^{7}\ \text{N/m}.$$

Jadi frekuensi natural sistem:

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{9{,}73\cdot10^{7}}{800}} = \underline{\underline{3{,}49\times10^{2}}}\ \text{rad/s}$$

GETARAN BEBAS TEREDAM ⑩



Newton ke II :

$$\Sigma F_{ext} = \Sigma F_{eff}.$$

$$mg - k(x + \Delta st) - c\dot{x} = m\ddot{x} \rightarrow mg - kx - k \cdot \underline{\underline{\Delta st}} - c\dot{x} = m\ddot{x}$$

Keseimbangan statik :

$$\Delta st \cdot k = m \cdot g$$

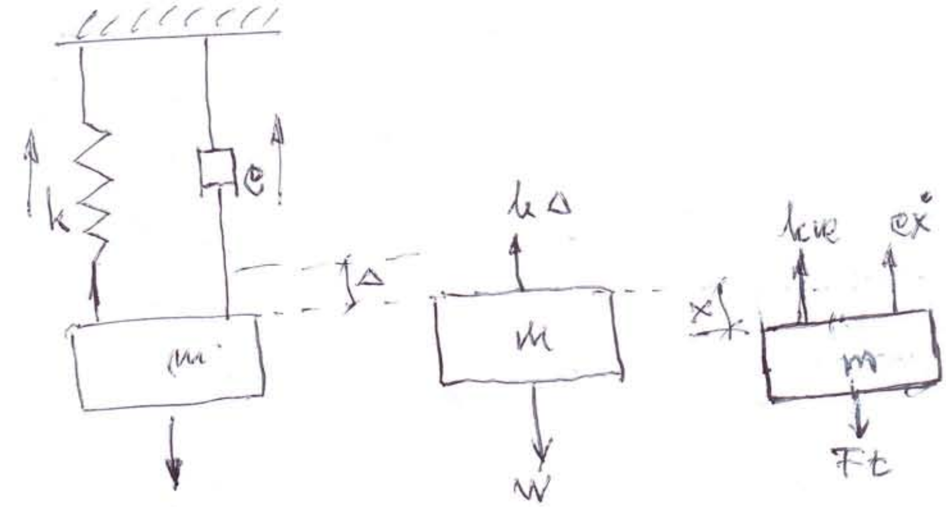
Persamaan getaran bebas SDOF menjadi

$$mg - kx - k \cdot \Delta st - c\dot{x} = m \cdot \ddot{x}$$

$$\cancel{mg} - kx - \cancel{m \cdot g} - c\dot{x} = m \cdot \ddot{x}$$

$$m\ddot{x} + \underline{\underline{c\dot{x}}} + kx = 0$$

GETARAN BEBAS TEREDAM



Persamaan kesetimbangan / homogen:

$m\ddot{x} + c\dot{x} + kx = 0$

$x = A \cdot e^{st}$ → A & s konstanta, penurunan thd waktu

$\dot{x} = \frac{d}{dt}(Ae^{st}) = sAe^{st}$

$\ddot{x} = \frac{d^2}{dt^2}(Ae^{st}) = s^2 Ae^{st}$

$m s^2 A e^{st} + c \cdot s \cdot A e^{st} + k A e^{st} = 0$

$(ms^2 + cs + k) Ae^{st} = 0 \quad : Ae^{st}$

$ms^2 + cs + k = 0$

$s_{1,2} = \dfrac{-c \pm \sqrt{c^2 - 4km}}{2m}$

$x = A_1 \cdot e^{s_1 t} + A_2 \cdot e^{s_2 t}$

$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$

$\omega_n^2 = \dfrac{k}{m} = \left(\dfrac{c_c}{2m}\right)^2 \rightarrow c_c = 2\sqrt{km}$ teoritis

$c_{teori} = 2m\omega_n$

$\frac{c}{m} = 2\zeta\omega_n$ ← $\frac{c}{m} = 2\omega_n\zeta$

$\zeta = \dfrac{c}{2\sqrt{km}} = \dfrac{c}{c_c}$

$c_c =$

$\zeta\omega_n = \dfrac{c}{2m} = \zeta\dfrac{c_c}{2m}$

$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$

$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = 0$

$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$s_{1,2} = \dfrac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2}$

$S_{1.2} = -\xi W_n \pm W_n \sqrt{\xi^2 - 1}$

$S_{1.2} = (-\xi \pm \sqrt{\xi^2 - 1}) W_n$

Tergantung nilai ξ :

① jika $\xi > 1$

$u = B_1 e^{-\xi W_n t + W_n \sqrt{\xi^2 - 1}} + B_2 e^{\ldots}$

② jika $\xi = 1$

$u = B_1 e^{-W_n t} + B_2 t e^{-W_n t}$

$= e^{-W_n t} (B_1 + B_2 t)$

$am^2 + bm + c = 0$

① Kedua akarnya riil & berbeda $m = m_1$; $m = m_2$

$y = A e^{m_1 x} + B e^{m_2 x}$

② Kedua akarnya riil & sama $m = m_1$

$y = e^{m_1 x} (A + Bx)$

③ Kedua akarnya komplek

$m = \alpha \pm j\beta$

$y = e^{\alpha x} (A \cos \beta x + B \sin \beta x)$

③ jika $\xi < 1$

$u = e^{-\xi W_n t} [B_1 \cos W_d t + B_2 \sin W_d t]$

$u = B e^{-\xi W_n t} \sin (W_d t + \psi)$

$B = \sqrt{B_1^2 + B_2^2}$; $\psi = tg^{-1} \left(\frac{B_1}{B_2} \right)$

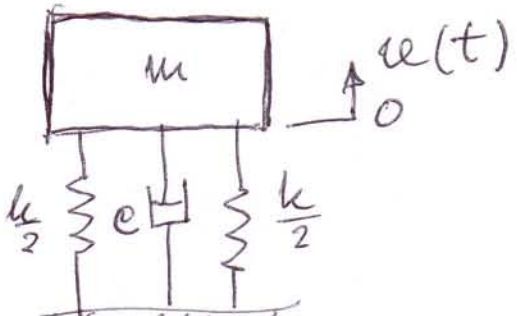
$W_d = W_n \sqrt{1 - \xi^2}$

W_d : frekwensi getaran teredam

ξ : faktor redaman

Contoh:

① Suatu massa 25 kg, kekakuan pegas 10 kN/m, redaman 140 Ns/m. Sistem mula² dalam keadaan diam pada kondisi statiknya dan kemudian massa diberi kecepatan awal 100 m/s. Lihat gambar. Hitung: kecepatan dan perpindahan dlm fungsi waktu



Jawab:

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{140}{2\sqrt{10\cdot 10^3 \cdot 25}} = 0,14$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10^4}{25}} = 20 \text{ rad/s}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 20\sqrt{1-0,14^2} = 19,8 \text{ rad/s}$$

$$\zeta\cdot\omega_n = 0,14\cdot 20 = 2,8$$

Dari (3):

$$x = e^{-\zeta\omega_n t}(B_1\cos\omega_d t + B_2\sin\omega_d t)$$

$$x = \underbrace{e^{-2,8t}}_{u}\,\underbrace{(B_1\cos 19,8t + B_2\sin 19,8t)}_{v}$$

$$x' = u'v + v'u$$

$$\dot{x} = -2,8e^{-2,8t}(B_1\cos 19,8t + B_2\sin 19,8t)$$
$$+\, e^{-2,8t}\cdot 19,8(-B_1\sin 19,8t + B_2\cos 19,8t)$$

Kondisi awal $x(0)=0$; $t=0$

$$x(0) = e^0(B_1\cdot\cos 0 + B_2\cdot\sin 0)$$
$$0 = 1\cdot(B_1+0) \rightarrow \underline{B_1=0}$$

Kondisi awal $\dot{x}(0) = 100$ m/s ; $t=0$

$$\dot{x}(0) = -2,8\cdot e^0(0\cdot\cos 0 + B_2\sin 0) + 19,8\cdot e^0(-0\cdot 0 + B_2\cdot e$$
$$100 = 0 + 0 + 19,8\cdot 1\cdot B_2$$
$$B_2 = \frac{100}{19,8} = \underline{\underline{5,05}}$$

Jadi respon posisi dan kecepatan massa:

(a) $x = e^{-2,8t}(0 + 5,05\sin 19,8t) = 5,05e^{-2,8t}\sin 19,8t$

(b) $\dot{x} = -2,8e^{-2,8t}(0+5,05\sin 19,8t) + 19,8\cdot e^{-2,8t}(0+5,05\cos 19$

$$\dot{x} = -2,8e^{-2,8t}(5,05\sin 19,8t) + 19,8e^{-2,8t}(5,05\cos 19,8t)$$

$$\dot{x} = e^{-2,8t}(\overset{B_1}{-14,14}\sin 19,8t + \overset{B_2}{100}\cos 19,8t)$$

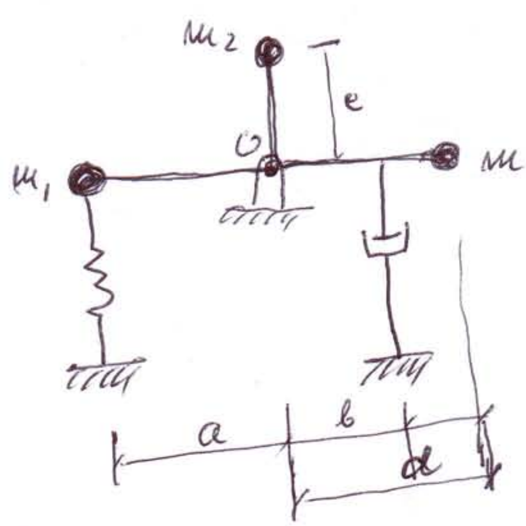
$$\dot{x} = Be^{-\zeta\omega_n\cdot t}\sin(\omega_d t + \psi)$$
$$= 101\cdot e^{-2,8t}\cdot\underline{\underline{\sin(19,8t - 0,141)}}$$

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(-14,14)^2 + 10}$$
$$= 101$$

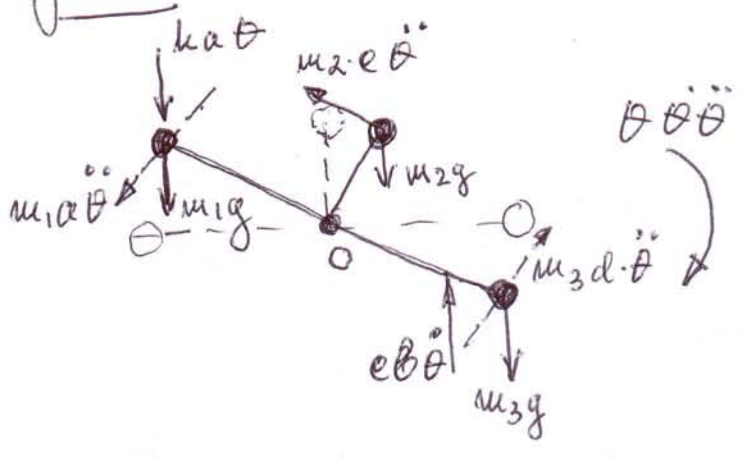
$$\psi = tg^{-1}\frac{B_1}{B_2} = tg^{-1}\frac{-14,14}{100}$$
$$= -0,1414$$

(2)

Suatu sistem yg bergetar. Massa batang diabaikan, turunkan persamaan gerak dan frekwensi pribadi sistem.



Jawab -



$$\Sigma M_O = 0$$

$$J_O \ddot{\theta} = 0$$

$$m_1 \cdot a \cdot \ddot{\theta} + m_2 \cdot e \cdot \ddot{\theta} + m_3 d \cdot \ddot{\theta} = m_2 g(e\theta) - (cb\dot{\theta})(b) - (ka\theta)(a$$

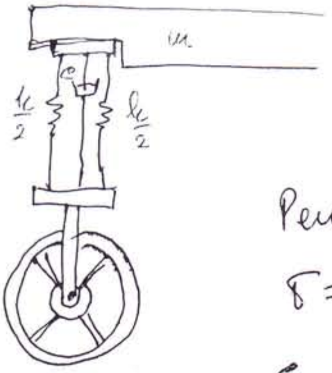
$$\cdot a \qquad \cdot e \qquad \cdot d$$

$$\underbrace{(m_1 \cdot a^2 + m_2 e^2 + m d^2)}_{m} \ddot{\theta} + \underbrace{cb^2}_{c} \dot{\theta} + \underbrace{(ka^2 - m_2 g e)}_{k} \theta = 0$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{ka^2 - m_2 g e}{m_1 a^2 + m_2 e^2 + m d^2}}$$

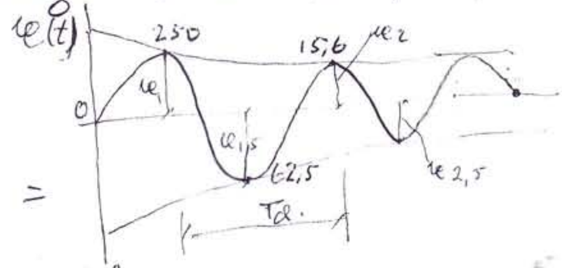
(15)

Suatu massa $m = 200$ kg, periode redaman $T_d = 2$ detik, amplitudo $A = 250$ mm, Amplitudo berikutnya karena ada redaman tersisanya 1/4 nya atau $x_{1,5} = \frac{x_1}{4}$.
Tentukan konstanta pegas (k), konstanta redaman (c) dan kecepatan awal yang menghasilkan amplitudo maksimum 250 mm.



Jawab

$\xi = \quad =$



Penurunan logaritmik:

$$\delta = \ln\left(\frac{x(t)}{x(t+T_d)}\right) = \ln\frac{x_1}{x_2} = \ln\left(\frac{250}{15{,}6}\right) = \ln(16) = 2{,}7726$$

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} = \frac{2{,}7726}{\sqrt{4\cdot\pi^2 + 2{,}7726^2}} = 0{,}4037$$

Redaman getaran:

$$\omega_n = \frac{2\pi}{T_d\sqrt{1-\xi^2}} = \frac{2\pi}{2\sqrt{1-(0{,}4037)^2}} = 3{,}4338 \text{ rad/s}$$

Konstanta pegas:

$$k = m\cdot\omega_n^2 = 200.(3{,}4338)^2 = 2358{,}2652 \text{ N/m}$$

Konstanta redaman kritis:

$$c_c = 2m\sqrt{\frac{k}{m}} = 2m\,\omega_n = 2\cdot 200\cdot\sqrt{\frac{2358{,}2652}{200}} = 1373{,}54 \text{ Ns/m}$$

Konstanta redaman sistem:

$$c = \xi . c_c = 0{,}4037\cdot 1373{,}54 = 554{,}4981 \text{ Ns/m}$$

Jika diketahui displacement massa maksimum pada t_1 maka

1 rad = 57,32°

asin 0,9149 = 66,19°

hal kebalikannya dimulai

$$\sin\omega_d . t_1 = \sqrt{1-\xi^2}$$

$$\sin\omega_d . t_1 \approx \sin\pi t_1 = \sqrt{1-0{,}4037^2} = 0{,}9149$$

$$t_1 = \frac{\sin^{-1}(0{,}9149)}{\pi} \approx 0{,}3678\text{ s} \rightarrow \frac{66{,}19/57{,}32}{\pi} = 0{,}3678\text{ s}$$

atau $= \frac{\sin^{-1}(0{,}9149)\,(\text{rad})}{\pi} = 0{,}3678$ s

Persamaan perpindahan yang melintasi titik tengah atau posisi netral getaran

$$x(t) = Ae^{-\xi\omega_n\cdot t}.\sin\omega_d\cdot t = A\cdot e^{-\xi.\omega_n\cdot t}\sqrt{1-\xi^2}$$

$$0{,}25 = A\cdot e^{-(0{,}4037).(3{,}4338)(0{,}3678)}\sqrt{1-(0{,}4037)^2}$$

∴ Amplitudo max $A = 0{,}455$ m.

Persamaan kecepatan penurunan dari hal. sebelumnya: (16)

$$\dot{x}(t) = A \cdot e^{-\xi \cdot \omega_n \cdot t} (-\xi \omega_n \cdot \sin \omega_d t + \omega_d \cdot \cos \omega_d t)$$ sama

Kecepatan awal $\dot{x}(t=0) = x_0$ saat amplitudo max:

$$\dot{x}(t=0) = x_0 = A\omega_d = A \cdot \omega_n \sqrt{1-\xi^2}$$ → dari $A e^{-\xi \omega_n t} \cdot \sin \omega_d t$

$$= 0{,}455 \cdot 3{,}4338 \sqrt{1-0{,}4037^2}$$

$$= \underline{1{,}4294}\ m/s$$

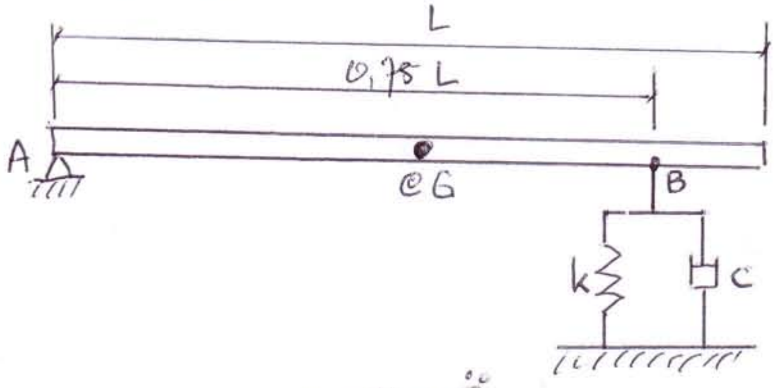
$$\dot{x} = -A \xi \omega_n e^{-\xi \omega_n t} \cdot \sin \omega_d t + A e^{-\xi \omega_n t} \cdot \omega_d \cos \omega_d t$$

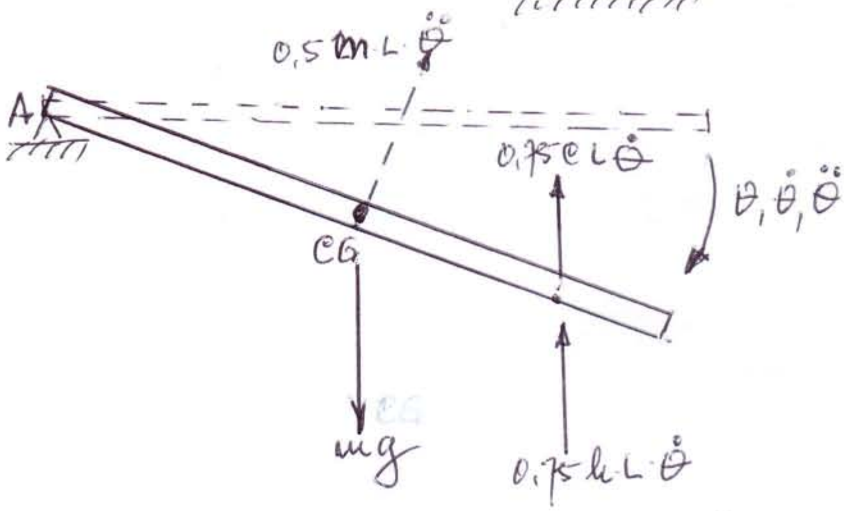
$$\dot{x} = -A \xi \omega_n \cdot 1 \cdot 0 + A \cdot 1 \cdot \omega_d$$

$$\dot{x} = A \cdot \omega_d .$$

○ Sebuah balok AB dengan letak pusat massa (CG) & massa m ditumpu dua buah pegas dan tumpuan di O jika $k_1 = 2k$ dan? $k_2 = k$, turunkan persamaan gerak sistem dan frekuensi pribadi sistem jika $J_{CG} = \frac{1}{12} m L^2$.

k dan c





Momen akibat gaya statik : 0,5 L cos θ diabaikan

$$(0{,}75\, c\, L\dot{\theta})(0{,}75L) + (0{,}75\, k\, L\theta)(0{,}75L) = $$

$$J_{CG}\,\ddot{\theta} + (0{,}5\, m L\ddot{\theta})(0{,}5L) + (0{,}75\, c\, L\dot{\theta})(0{,}75L) + (0{,}75\, k\, L\theta)(0{,}75L) = 0$$

$$\frac{1}{12} m L^2 \ddot{\theta} + 0{,}25\, m L^2 \ddot{\theta} + 0{,}5625\, c\, L^2 \dot{\theta} + 0{,}5625\, k L^2 = 0$$

$$\frac{1}{3} m L^2 \ddot{\theta} + \frac{9}{16} c L^2 \dot{\theta} + \frac{9}{16} k L^2 \theta = 0$$

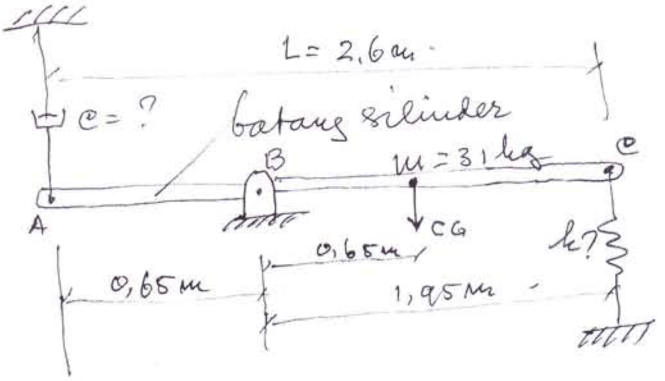
∴ Persamaan geraknya : $\underline{\frac{1}{3} m \ddot{\theta} + \frac{9}{16} c \dot{\theta} + \frac{9}{16} k \theta = 0}$

Kekakuan ~~mass~~ equivalen:

$$k_{eq} = \frac{9}{16} k \;;\; m_{eq} = \frac{1}{3} m$$

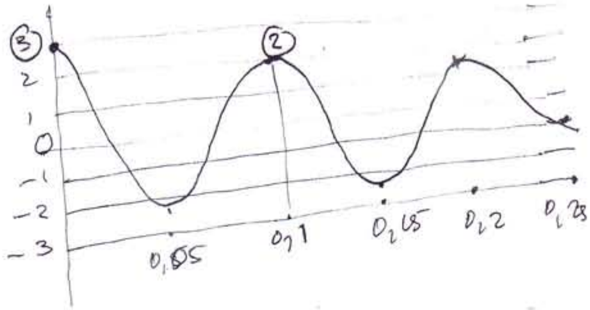
Frekuensi pribadi sistem :

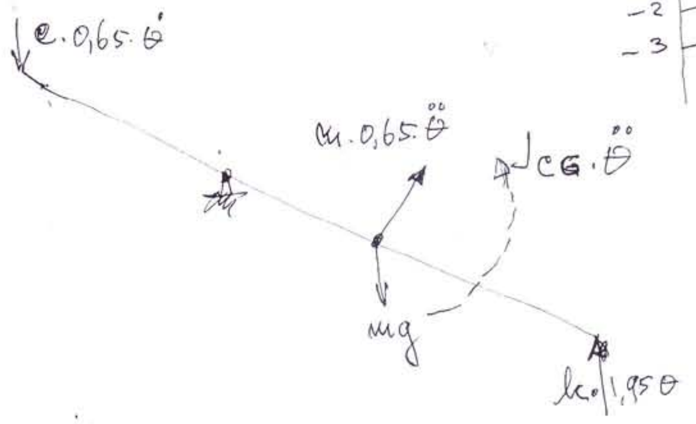
$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{27 k}{16 m}}$$



$J_{CG} = \frac{1}{12} mL^2$

Dari osciloscope diketahui:





$$J_{CG}.\ddot{\theta} + m.0{,}65.\ddot{\theta}.0{,}65 + c.0{,}65.\dot{\theta}.0{,}65 + k.1{,}95.\theta.1{,}95 = 0$$

$$J_{CG}.\ddot{\theta} + m.0{,}65^2\ddot{\theta} + c.0{,}65^2.\dot{\theta} + k.1{,}95^2.\theta = 0$$

$$\frac{1}{12} m.2{,}6^2.\ddot{\theta} + m.0{,}65^2\ddot{\theta} + c.0{,}65^2.\dot{\theta} + 3{,}8025\,k.\theta = 0$$

$$m\left(\frac{2{,}6^2}{12} + 0{,}65^2\right)\ddot{\theta} + 0{,}4225\,c\,\dot{\theta} + 3{,}8025\,k\,\theta = 0$$

$$m.0{,}9858\,\ddot{\theta} + 0{,}4225\,c\,\dot{\theta} + 3{,}8025\,k\,\theta = 0$$

$$m.\ddot{\theta} + 0{,}4285\,c\,\dot{\theta} + 3{,}8573\,k\,\theta = 0$$

∴ Pers. gerak : $m.\ddot{\theta} + \underline{0{,}4285}\,c\,\dot{\theta} + 3{,}8573\,k\,\theta = 0$

Periode redaman dari getaran bebas diperoleh 0,1 s

$$\delta = \ln\left[\frac{x(0)}{x(0{,}1\,s)}\right] = \ln\left(\frac{3}{2}\right) = 0.405$$

Rasio redaman ξ:

$$\xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = \frac{0.405}{\sqrt{4\pi^2+0{,}405^2}} = 0{,}0643$$

Damping frekuensi & frekuensi natural

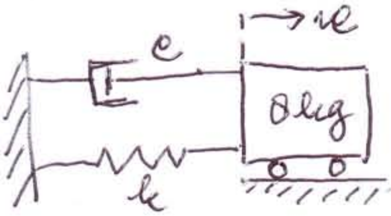
$$\omega_d = \frac{2\pi}{T_d} = \frac{6{,}28}{0{,}1} = 62{,}8 \text{ rad/s}$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{62{,}8}{\sqrt{1-0{,}0643^2}} = 62{,}96 \text{ rad/s}$$

Konstanta pegas (k) & konstanta redaman (c):

∴ $\omega_n^2 = \frac{k}{m} \rightarrow k = m.\omega_n^2 \rightarrow 3{,}8573\,k = 31.62{,}96^2 \Rightarrow$ ∴ $k = 31857$ N/m

∴ $c = 2m\xi.\omega_n \rightarrow 0{,}4285\,c = 2.31.0{,}0643.62{,}96 \rightarrow$ ∴ $c = 585{,}7$ Ns/m



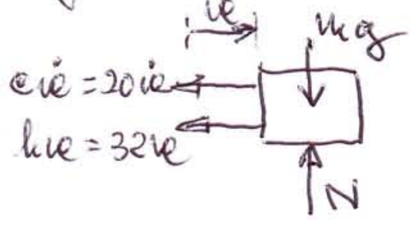
Benda dengan masa 8 kg dipindah kekanan 0,2 m dan dilepas pd waktu $t = 0$

Tentukan perpindahan $t = 2''$

Koefisien redaman $c = 20$ N det/m

Kekakuan pegas $k = 32$ N/m.

Jawab



$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{32}{8}} = 2 \text{ rad/s}$$

$$\xi = \frac{c}{2m\omega_n} = \frac{20}{2\cdot 8\cdot 2} = 0{,}625$$

Karena $\xi < 1$, maka termasuk getaran dg redaman subkritis

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 2\cdot\sqrt{1-0{,}625^2} = 1{,}561 \text{ rad/s}$$

Persamaan gerakan:

$$x = e^{-\xi\omega_n t}(B_1 \cos \omega_d t + B_2 \sin \omega_d \cdot t)$$

Kondisi awal $x = 0{,}2$ m $\rightarrow t = 0$

$$x = e^{-0{,}625\cdot 2\cdot 0}(B_1 \cdot \cos 0 + B_2 \sin 0)$$

$$x = B_1 + 0$$

$$\underline{0{,}2 = B_1}$$

Persamaan kecepatan:

$$\dot{x} = -\xi\omega_n \cdot e^{-\xi\omega_n t}(B_1 \cos \omega_d t + B_2 \sin \omega_d \cdot t) + e^{-\xi\omega t}(-\omega_d \cdot B_1 \sin \omega_d t + \omega_d B_2 \cdot \cos \omega_d t)$$

$$0 = -\xi\omega_n \cdot e^0(0{,}2 \cos 0 + B_2 \cdot \sin 0) + e^0(-\omega_d \cdot B_1 \sin 0 + \omega_d \cdot B_2 \cdot \cos 0)$$

$$0 = (-0{,}2 \cdot \xi\omega_n + 0) + (0 + \omega_d \cdot B_2 \cdot 1)$$

$$0 = -0{,}2 \cdot \xi\omega_n + 1{,}561 \cdot B_2$$

$$B_2 = \frac{0{,}2 \cdot 0{,}625 \cdot 2}{1{,}561} = \underline{0{,}1602}$$

Jadi $x_{t=2} = e^{-\xi\omega_n t}(0{,}2 \cos \overbrace{1{,}561 \cdot t}^{\text{radial}} + 0{,}1602 \sin \overbrace{1{,}561 \cdot t}^{\text{radial}}) = \underline{0{,}0162 \text{ m}}$

Ramsey

(20)

① B_1 dari x
B_2 dari $\dot{x}$ } $B_1 = 0$ B_2 = Ada.

B_1 & B_2 masuk ke x
B_1 & B_2 — " — $\dot{x}$

$\dot{x}$ diselesaikan → ketemu $B_{1\,baru}$ & $B_{2\,baru}$ → ketemu B φ.

$\dot{x} = B e^{-\xi \omega_n t} \sin(\omega_d t + \varphi)$.

② B_1 dari θ
B_2 dari $\dot{\theta}$
B_1 & B_2 masuk ke θ. → ketemu B & φ

$\theta = B e^{-\xi \omega_n t} \sin(\omega_d t + \varphi)$

$\dot{\theta} = \ldots$ dari ↑

Meriam

① $x = \underline{C e^{-\xi \omega_n t}} \, \underline{\sin(\omega_d t + \varphi)}$

$\dot{x} = u'v + uv'$

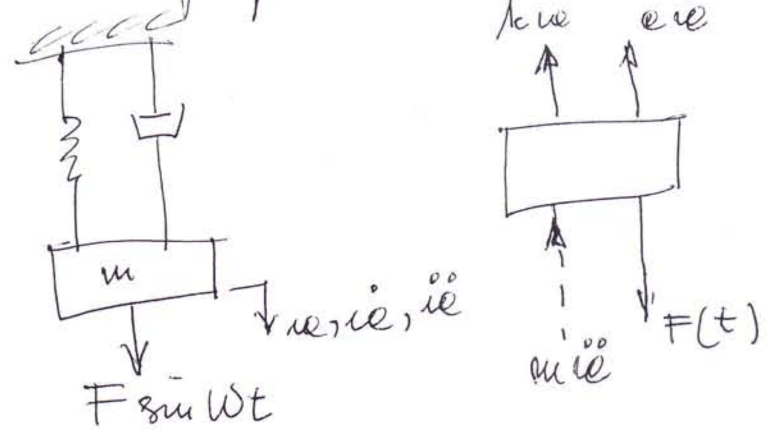
Timbul dua persamaan.
Dengan trial & error ketemu C & φ
Ketemu x & $\dot{x}$

② atau B_1 dari x
B_2 dari $\dot{x}$

Ketemu nilai $x = e^{-\xi \omega_n t} (B_1 \cos \omega_d t + B_2 \sin \omega_d t)$.

GETARAN PAKSA

Adalah sistem yang bergetar karena adanya gaya luar yang bekerja pada sistem tersebut. Gaya luar tersebut misalnya ketidak seimbangan mesin yang berputar, gaya² yang dihasilkan mesin torak. Gaya luar ini disebut Eksitasi.



Diperoleh

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

$$m\ddot{x} + c\dot{x} + kx = F \sin \omega t \quad \dots (1)$$

Solusi dalam keadaan steady:

$$x_p = X \sin(\omega t - \phi)$$

X : amplitudo

$$\dot{x}_p = \omega X \cos(\omega t - \phi)$$

ϕ : beda fase simpangan thd gaya eksitasi

$$\ddot{x}_p = -\omega^2 X \sin(\omega t - \phi)$$

Substitusikan ke (1)

$$-m\omega^2 X \sin(\omega t - \phi) + c\,\omega X \cos(\omega t - \phi) + kX \sin(\omega t - \phi) = F \sin \omega t$$

$$-m\omega^2 X \sin(\omega t - \phi) + kX \sin(\omega t - \phi) + c\,\omega X \cos(\omega t - \phi) = F \sin \omega t$$

$$(k - m\omega^2) X \sin(\omega t - \phi) + c\omega X \cos(\omega t - \phi) = F \sin \omega t$$

$$(k - m\omega^2) X (\sin \omega t \cdot \cos\phi - \cos \omega t \cdot \sin\phi) + c\omega X (\cos \omega t \cdot \cos\phi + \sin \omega t \cdot \sin\phi) = F \sin \omega t$$

$$kX \sin\omega t \cdot \cos\phi - kX \cos\omega t \cdot \sin\phi - m\omega^2 X \sin\omega t \cdot \cos\phi + m\omega^2 X \cos\omega t \cdot \sin\phi + c\omega X (\cos\omega t \cdot \cos\phi + \sin\omega t \cdot \sin\phi) = F \sin\omega t$$

$$\left[(k - m\omega^2)\cos\phi + c\omega \sin\phi\right] X \sin\omega t + \left[(-k + m\omega^2)\sin\phi + c\omega\cos\phi\right] X \cos\omega t = F \sin\omega t$$

untuk sembarang t:

$$\left[(k - m\omega^2)\cos\phi + c\omega\sin\phi\right] X \sin\omega t - F \sin\omega t = 0$$

$$\left[(m\omega^2 - k)\sin\phi + c\,\omega\cos\phi\right] = 0$$

catatan: $\cos^2\phi + \sin^2\phi = 1$

$$X = \frac{F}{(k - m\omega^2)\cos\phi + c\omega\sin\phi} = \frac{F}{\sqrt{(k - m\omega^2)^2\cos^2\phi + c^2\omega^2\sin^2\phi}}$$

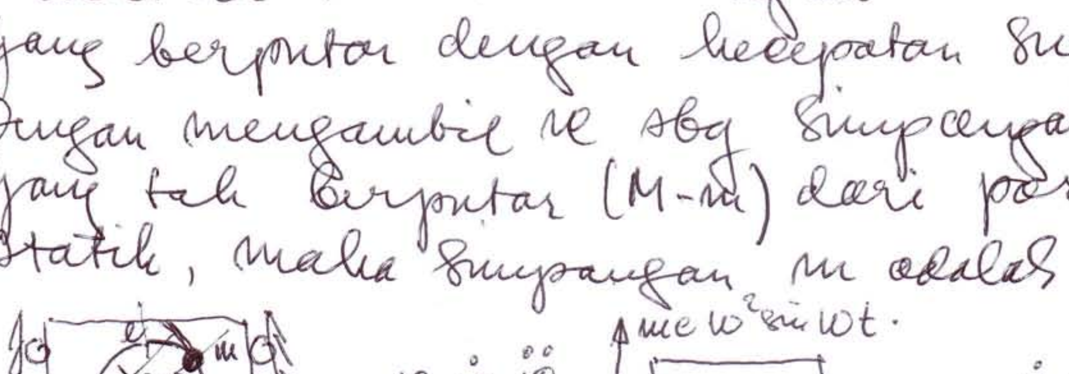
$$\therefore X = \frac{F}{\sqrt{(k - m\omega^2)^2 + c^2\omega^2}}$$

Massa Tak Seimbang

Ketidak seimbangan mesin² yang berputar merupakan sumber eksitasi getaran. Perhatikan gambar dibawah ini dimana hanya bergerak arah vertikal dan dirangsang oleh mesin yg berputar tidak seimbang.

Ketidak seimbangan ditunjukkan oleh oleh eksentrik massa m dengan eksentrisitas e yang berputar dengan kecepatan sudut ω.

Dengan mengambil x sbg simpangan massa yang tak berputar $(M-m)$ dari posisi seimbang statik, maka simpangan m adalah $x + e \sin \omega t$



Persamaan gerak adalah:

$$(M-m)\ddot{x} + m \frac{d^2}{dt^2}(x + e \sin \omega t) + c\dot{x} + kx = 0$$

$$(M-m)\ddot{x} + m \frac{d}{dt}(\dot{x} + e\omega \cos \omega t) + c\dot{x} + kx = 0$$

$$(M-\cancel{m})\ddot{x} + \cancel{m\ddot{x}} - me\omega^2 \sin \omega t + c\dot{x} + kx = 0$$

$$M\ddot{x} + c\dot{x} + kx = me\omega^2 \sin \omega t$$

$F_{eq} = me\omega^2$ = gaya pemaksa.

$$M\ddot{x} + c\dot{x} + kx = F_{eq} \cdot \sin \omega t$$

Dengan cara yg sama didapat
Amplitudo (X):

$$X = \frac{F_{eq}}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}} = \frac{me\omega^2}{\sqrt{(k-m\omega^2)^2 + c^2\omega^2}}$$ atau

$$X = \frac{me\omega^2}{k} \cdot R \qquad R = \frac{1}{2\zeta}$$

Dalam bentuk non dimensional $r = \frac{\omega}{\omega_n}$; $\omega^2 = \frac{k}{m}$, ma

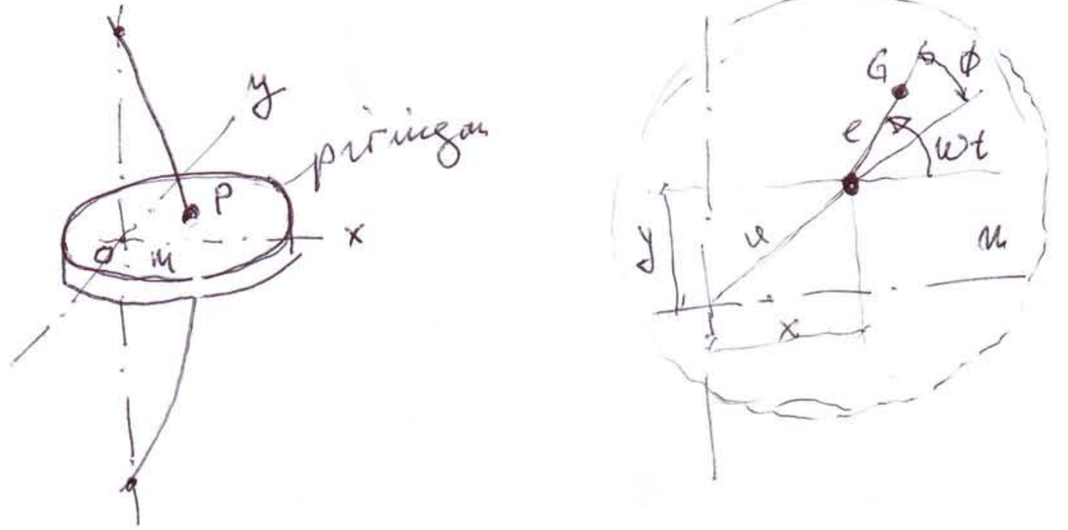
$$\frac{M}{m} \cdot \frac{X}{e} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta \frac{\omega}{\omega_n}\right]^2}}$$

$$\frac{M}{m} \cdot \frac{X}{e} = r^2 \cdot R \qquad R = \frac{1}{2\zeta}$$

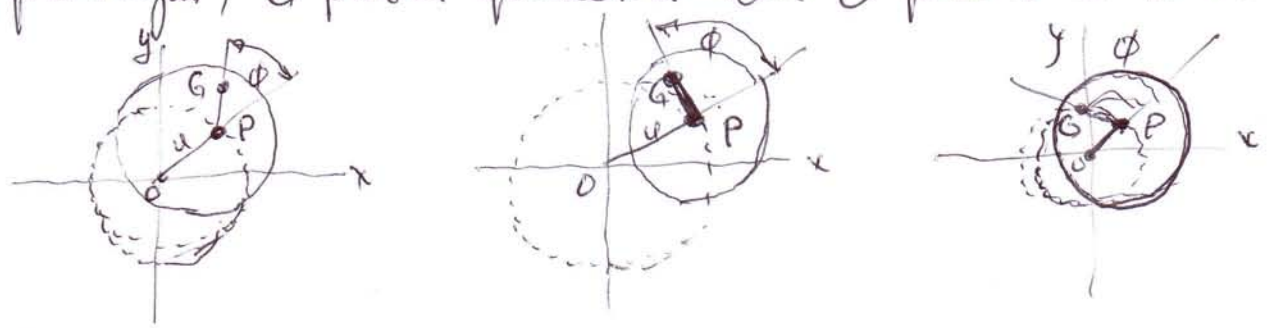
$$= \frac{r^2}{\sqrt{\left[(1-r)^2\right]^2 + (2\zeta r)^2}} \qquad X = \frac{me}{(2\zeta M)}$$

Kecepatan Kritis Poros

Dalam aplikasi mekanik bahwa masalah getaran yang ditimbulkan adalah oleh sistem poros dengan piringan yg tak seimbang. Kecepatan kritis terjadi pada saat kecepatan rotasi poros = frekuensi pribadi poros dlm arah lateral.



Piringan berputar dengan massa m, G pusat massa piringan, G pusat geometri dan O pusat rotasi.



Dengan menguraikan gaya ke sb x & y:

$$m\frac{d^2}{dt^2}(x + e\cos\omega t) = -kx - cx$$

$$m\frac{d^2}{dt}(y + e\sin\omega t) = -ky - c\dot{y}$$

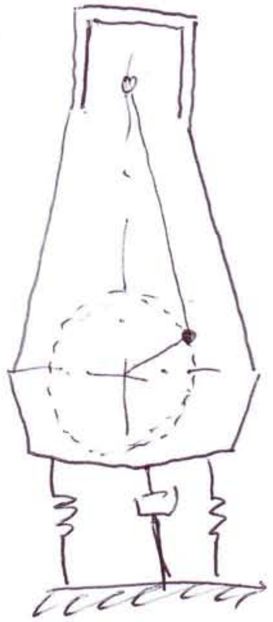
$$m\ddot{x} + c\dot{x} + kx = me\omega^2 \cdot \cos\omega t = F_{eq}\cos\omega t$$

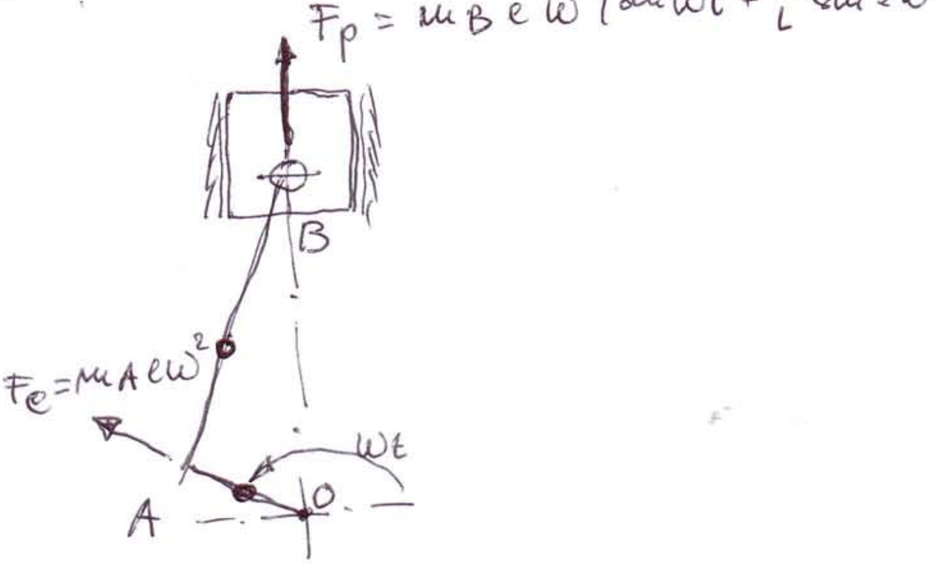
$$m\ddot{y} + c\dot{y} + ky = me\omega^2 \cdot \sin\omega t = F_{eq}\sin\omega t$$

Analog: $$X = Y = \frac{F_{eq}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} = \frac{me\omega^2}{k} R.$$

$$\frac{u}{e} = r^2 R = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

Getaran mesin torak





Gaya² yg bekerja pada torak:

$$F_p = m_B e \omega^2 (\sin \omega t + \frac{e}{L} \sin 2\omega t)$$

Gaya engkol:

$$F_e = m_A \cdot e \omega^2$$

Jika gaya engkol telah diseimbangkan maka gaya equivalen pada sistem adalah hanya gaya inersia torak

$$F_{eq} = m_B e \omega^2 (\sin \omega t + \frac{e}{L} \sin 2\omega t)$$

Maka persamaan geraknya dari sistem:

$$m\ddot{x} + c\dot{x} + kx = m_B e \omega^2 (\sin \omega t + \frac{e}{L} \sin 2\omega t)$$

Respon dalam keadaan stedi dg mensuperposisikan akibat gaya primer $m_B e \omega^2 \sin \omega t$ dan gaya sekunder $\frac{e}{L} m_B e \omega^2 \sin 2\omega t$).

Jika $x_p(t)$ adalah respon akibat gaya primer maka $x_p(t) = X_p \sin(\omega t - \phi_p)$ dimana

$$X_p = \frac{F_{eq}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} = \frac{m_B e \omega^2}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$\phi_p = -tg^{-1}\frac{\omega c}{k-\omega^2 m}$$

jika $x_s(t)$: respon akibat gaya skunder maka:

$$x_s(t) = X_s \sin(2\omega t - \phi_s)$$

dimana:

$$X_s = \frac{(e/L)\, m_B e \omega^2}{\sqrt{(k-m(2\omega)^2)^2 + (c(2\omega))^2}} \qquad \phi_s = -tg^{-1}\frac{2\omega c}{k-4\omega^2 m}$$

$$= \frac{m_B e^2 \omega^2}{L\sqrt{(k-4m\omega^2)^2 + (2c\omega)^2}}$$

$$x(t) = x_p(t) + x_s(t)$$

$$= X_p \sin(\omega t - \phi_p) + X_s \sin(2\omega t - \phi_s).$$

Contoh.

Suatu mesin torak dengan massa ekuivalen torak $m_B = 2$ kg dan massa total mesin 30 kg, kekakuan $k = 180$ kN/m, redaman $c = 300$ Ns/m, jari² engkol $e = 0{,}07$ m, panjang conecting rod $L = 0{,}28$ m.

Hitung respons sistem dalam fungsi frekuensi ω.

Jawab: ~~Amplitudo~~ respon primer:

$$X_p = \frac{m_B e \omega^2}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$X_p = \frac{2 \cdot 0{,}07\, \omega^2}{\sqrt{(1{,}8 \times 10^5\ N/m - (300\ kg)\omega^2)^2 + ((300\ Ns/m)\omega)^2}}$$

$$\phi_p = -tg^{-1} \frac{\omega (300\ Ns/m)}{1{,}8 \cdot 10^5\ N/m - (300\ kg)\omega^2} \rightarrow \text{atau } \phi_p = -tg^{-1} \frac{\omega . c}{k - \omega^2 m}$$

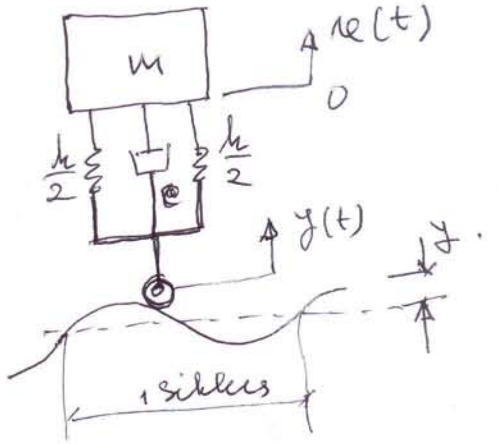
Amplitudo respon sekunder:

$$X_s = \frac{m e \cdot \omega^2}{L\sqrt{(k - 4m\omega^2)^2 + (2c\omega)^2}}$$

$$= \frac{2 \cdot 0{,}07\, \omega^2}{0{,}28\sqrt{(1{,}8 \cdot 10^5\ N/m - (120\ kg)\omega^2)^2 + ((600\ Ns/m)\omega)^2}}$$

$$\phi_s = -tg^{-1} \frac{\omega (600\ Ns/m)}{1{,}8 \cdot 10^5\ N/m - (120\ kg)\omega^2} \rightarrow \phi_s = -tg^{-1} \frac{2\omega . c}{k - 4\omega^2 m}$$

Sistem Suspensi Kendaraan.



Asumsi :

1. Kendaraan dibatasi shg merupakan sistem dg satu derajat kebebasan dlm arah vertikal.
2. Kekakuan roda dianggap tak terhingga shg ketidakrataan jalan langsung ditransmisikan ke sistem suspensi.
3. Roda bergerak mengikuti permukaan jalan yang dianggap sinusoidal.

Jika kondisi jalan merupakan fungsi sinusoidal L m/siklus, dan kecepatan kendaraan adalah V km/h, maka frekuensi eksitasi adalah :

$$f = \frac{v}{3600} \cdot \frac{1}{L} \quad Hz.$$

atau

$$\omega = 2\pi \frac{v}{3600} \cdot \frac{1}{L} \cdot rad/s.$$

Contoh.

Suatu trailer dengan massa dalam keadaan penuh 1200 kg dan beban kosong 300 kg. Konstanta pegas 500 kN/m. Faktor redaman $\xi = 0,4$ pada beban penuh. Kecepatan trailer 72 km/h. Kondisi jalan sinusiodal dg 4m/siklus.

Hitung: rasio amplitudo dalam keadaan penuh dan dalam keadaan kosong

Frekuensi eksitasi:

$$\omega = 2\pi \frac{v}{3600} \cdot \frac{1}{L} \text{ rad/s}$$

$$= 2\pi \frac{72}{3600} \cdot \frac{1}{4} = \underline{31,4} \text{ rad/s}$$

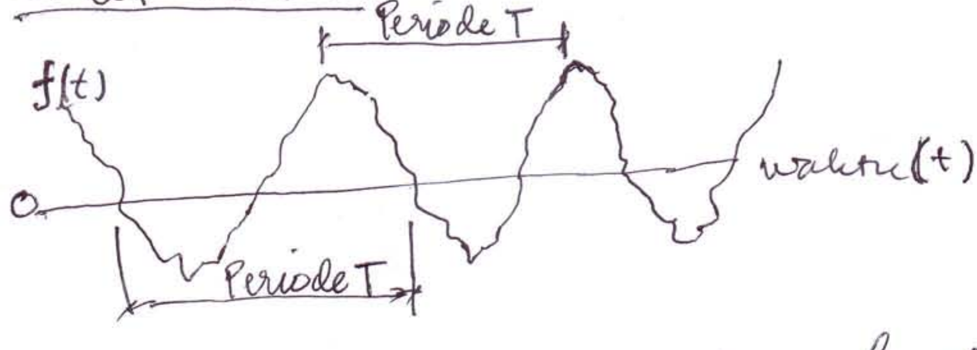
Koefisien redaman $c = 2\xi\sqrt{km}$, karena c dan k mempunyai nilai tetap maka ξ merupakan fungsi m. Maka faktor redaman dalam keadaan penuh adalah:

$$\xi_{full} = \xi_{kosong} \frac{\sqrt{m\ penuh}}{\sqrt{m\ kosong}} = 0,4 \frac{\sqrt{1200}}{\sqrt{300}} = 0,8$$

Frekuensi pribadi	Beban Penuh	Beban kosong
$\omega_n = \sqrt{\frac{k}{m}}$	$\omega_n = \sqrt{\frac{500\,000}{1200}} = 20,41$ rad/s	$\omega_n = \sqrt{\frac{500\,000}{300}} = 40,82$ rad/s
$r = \frac{\omega}{\omega_n}$	$r = \frac{31,4}{20,41} = 1,53$	$r = \frac{31,4}{40,82} = 0,769$
$\frac{X}{Y} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$	$\frac{X}{Y} = \frac{\sqrt{1+(2\cdot 0,4 \cdot 1,53)^2}}{\sqrt{(1-1,35^2)^2+(2\cdot 0,4\cdot 1,53)^2}} = 0,8706$	$\frac{X}{Y} = \frac{\sqrt{(1+(2\cdot 0,8\cdot 0,769)^2}}{\sqrt{(1-0,769)^2+(2\cdot 0,8\cdot 0,769)^2}} = 1,223$

Respon Terhadap Eksitasi Periodik.

Deret Forier.



Fungsi f(t) adalah periodik tetapi bukan harmonis

Periode T dinyatakan dalam bentuk deret:

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega_T \cdot t + b_n \sin n\omega_T t).$$

$$\omega_T = \frac{2\pi}{T}$$

a_0, a_n & b_n untuk fungsi F(t) diperoleh dari:

$$a_0 = \frac{2}{T}\int_0^T F(t)\,dt.$$

$$a_n = \frac{2}{T}\int_0^T F(t)\cos n\omega_T \cdot t \cdot dt \qquad n = 1, 2, \ldots$$

$$b_n = \frac{2}{T}\int_0^T F(t)\sin n\,\omega_T \cdot t \cdot dt \qquad n = 1, 2, \ldots$$

Jika gaya periodik F(t) dikenakan pd sistem dg satu derajat kebebasan, input gaya dg sejumlah n gaya harmonik, maka:

$$m\ddot{x} + c\dot{x} + kx = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega_T t + b_n \sin n\omega_T t)$$

Respon steady akibat tiap² komponen gaya eksitasi:

$$x = \frac{a_0}{2k} + \sum_{n=1}^{\infty} \frac{a_n \cos(n\omega_T - \phi_n)t + b_n \sin(n\omega_T - \phi_n)t}{k\sqrt{(1-n^2r^2)^2 + (2\xi n r)^2}}$$

$$\phi_n = tg^{-1}\frac{2\xi n r}{1-n^2 \cdot r^2}$$

Contoh:

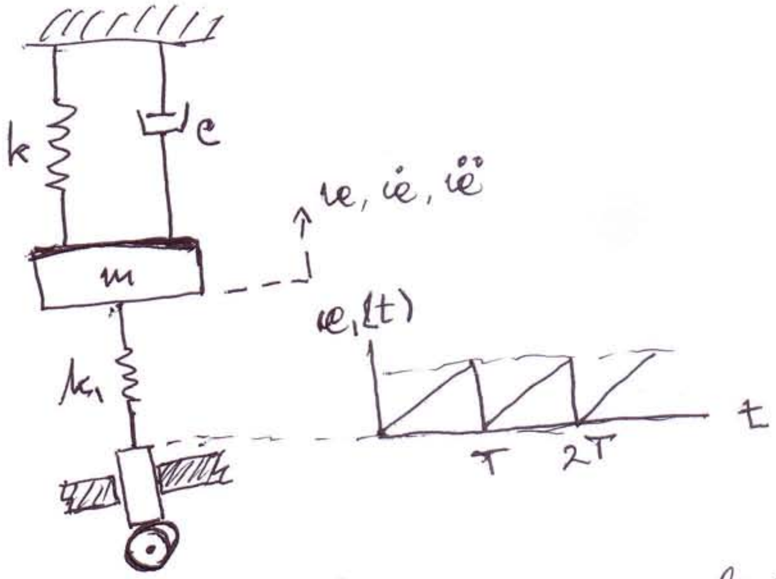
Pada gambar dibawah ini adalah suatu cam menggera[k] suatu sistem massa-pegas.

Jika maksimum $x_1(t) = 20$ mm; kecepatan sudut cam 90 ; massa 25 kg; $k_1 = k_2 = 6$ kN/m; koefisien redaman $c = 0{,}2$ kNs/m.

Hitung:

Respon $x_k(t)$ dalam keadaan stedi

Jawab



Uraikan dahulu eksitasi menurut deret Fourier:

$x_1(t) = \frac{1}{T} t$ untuk $0 \le t \le T$

$\omega_T = 90 \frac{2\pi}{60} = 3\pi \rightarrow \omega_T = \frac{2\pi n}{60}$

$T = \frac{2\pi}{\omega_T} = \frac{2\pi}{3\pi} = \frac{2}{3}$ s, maka

$x_1(t) = \frac{|X_1| t}{T} = \frac{3}{2} |X_1| t = \frac{3}{2} \cdot 20t = 30t$ untuk $0 \le t \le \frac{2}{3}$

Menentukan koefisien Fourier:

$a_0 = \frac{2}{T} \int_0^T F(t)\, dt = \frac{2}{\frac{2}{3}} \left[\int_0^{2/3} 30t\, dt \right] = 45t^2 \Big]_0^{2/3} = 20$

$a_1 = \frac{2}{T} \int_0^T F(t) \cos \omega_T t\, dt = 3 \left[\int_0^{2/3} 30t \cos 3\pi . t\, dt \right] = 0 = a_2 = a_3 \cdots$

$b_n = \frac{2}{T} \int_0^T F(t) \sin n \omega_T \cdot t \cdot dt \qquad n = 1, 2, \cdots$

Koefisien untuk deret sinus:

$$b_1 = \frac{2}{T}\int_0^{2/3} 30t \sin 3\pi t \cdot dt = 3\int_0^{2/3} 30t \sin 3\pi t \, dt = -\left(\frac{20}{\pi}\right)$$

$$b_1 = \int_0^{2/3} \underbrace{3 \cdot 30t}_{u} \cdot \underbrace{\sin 3\pi t \cdot dt}_{dv} = uv - \int_0^{} v\,du.$$

$$= 90t \cdot \frac{-\cos 3\pi t}{3\pi} + \int_0^{2/3} \frac{\cos 3\pi t}{3\pi} \cdot 90t \, dt.$$

$$= \left[90t \cdot \frac{-\cos 3\pi t}{3\pi} + \left(\frac{90}{3\pi} \cdot \frac{\sin 3\pi t}{3\pi}\right)\right]_0^{2/3}$$

$$= \left[\frac{90 \cdot \frac{2}{3}}{3\pi} \cdot \left(-\cos 3\pi \cdot \frac{2}{3}\right) + \left(\frac{90}{3^2\pi^2} \cdot \left(\sin 3\pi \cdot \frac{2}{3}\right)\right)\right.$$

$$\left. - \frac{90 \cdot 0}{3\pi} \cdot \left(-\cos 3\pi \cdot 0\right) - \left(\frac{90 \cdot 0}{3^2\pi} \cdot \left(\sin 3\pi \cdot 0\right)\right)\right]$$

$$= -\frac{180}{9\pi} \cdot \cos 2\pi - 0 - 0\left[0 - 0 - 0\right)\Big]$$

$$= -\left(\frac{20}{\pi}\right)$$

$$b_2 = \frac{2}{T}\int_0^{2/3} 30t \cdot \sin 6\pi t \, dt = \int_0^{\frac{2}{3}} 3 \cdot 30t \cdot \sin 6\pi t \, dt$$

$$= 90t \cdot \frac{-\cos 6\pi t}{6\pi} + \int_0^{2/3} \frac{\cos 6\pi t}{6\pi} \cdot 90t \, dt.$$

$$= -\frac{90t}{6\pi} \cdot \cos 6\pi t + \frac{90}{6\pi} \cdot \sin \frac{6\pi t}{6\pi}\Bigg]_0^{2/3}$$

$$= -\frac{90 \frac{2}{3}}{6 \cdot \pi} \cdot \cos 6\pi \frac{2}{3} + \frac{90}{6^2\pi^2} \cdot \sin 6\pi \, {}^2/_3 - 0 -$$

$$= -\frac{180}{18\pi} \cdot \cos 4\pi - 0 - 0 - 0$$

$$= -\frac{20}{2\pi}$$

$$b_n = -\frac{20}{n\pi}$$

=0

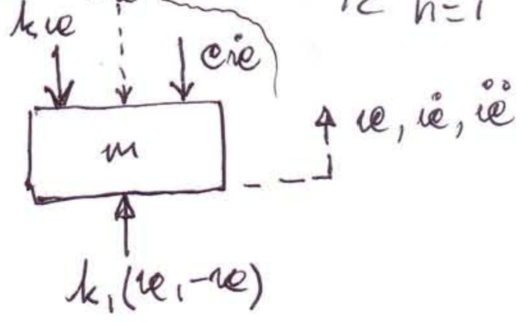
Dengan menggunakan deret Fourier:

$$x_1(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega_T t + b_n \sin n\omega_T t)$$

$$= \frac{20}{2} + b_n \cdot \sin n\omega_T t + \cdots$$

$$= 10 - \frac{20}{\pi} \sin 3\pi t - \frac{20}{2\pi} \sin 6\pi t - \frac{20}{3\pi} \sin 9\pi t - \cdots$$

$$= 10 - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{20}{n} \sin 3n\pi t .$$



Persamaan gerak dari diagram benda bebas:

$$m\ddot{x} + c\dot{x} + (k+k_1)x = kx_1(t) = k_1\left(10 - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{20}{n}\sin 3n\pi t\right)$$

Respons akibat eksitasi konstan:

$$x_0 = \frac{10k_1}{(k+k_1)}$$

Respons dari eksitasi frekuensi harmonik $n\omega$:

$$|X_n| = \frac{-20k_1}{n\pi} \frac{1}{(k+k_1)\sqrt{(1-n^2r^2)^2 + (2\xi n r)^2}}$$

$$= - \frac{20k_1}{n\pi(k+k_1)\sqrt{(1-n^2r^2)^2 + (2\xi n r)^2}}$$

atau $x_n = |X_n| \sin(3n\pi t - \phi_n)$

dimana:

$$\phi_n = tg^{-1}\left(\frac{2\xi n r}{1-n^2r^2}\right)$$

Respons sistem:

$$x(t) = x_0 + \sum_{n=1}^{n} x_n = \frac{10k_1}{(k+k_1)} - \sum_{1}^{n} |X_n| \sin(3n\pi t - \phi_n)$$

$$= \frac{10k_1}{(k+k_1)} - \sum_{1}^{n} \frac{20k_1}{n\pi(k+k_1)\sqrt{(1-n^2r^2)^2 + (2\xi n r)^2}}$$

Dari data didapat :

$$\omega_n = \sqrt{\frac{k+k_1}{m}} = \sqrt{\frac{1{,}2 \cdot 10^4}{25}} = 21{,}9 \ \text{rad}/s$$

$$r = \frac{\omega_T}{\omega_n} = \frac{3\pi}{\omega_n} = \frac{3\pi}{21{,}9} = 0{,}43.$$

$$\xi = \frac{c}{2\sqrt{(k+k_1)m}} = \frac{200}{2\sqrt{1{,}2 \cdot 10^4 \cdot 25}} = 0{,}1826$$

$$\phi_n = tg^{-1} \frac{2\xi n r}{1-n^2r^2} = tg^{-1} \frac{0{,}157\, n}{1-0{,}1849\, n^2}$$

$$x_p(t) = 5 - \sum_1^n \frac{100}{n\pi\sqrt{(1-0{,}1849\, n^2)^2 + (0{,}157 \cdot n)^2}} \sin(3n\pi t - \phi_n$$