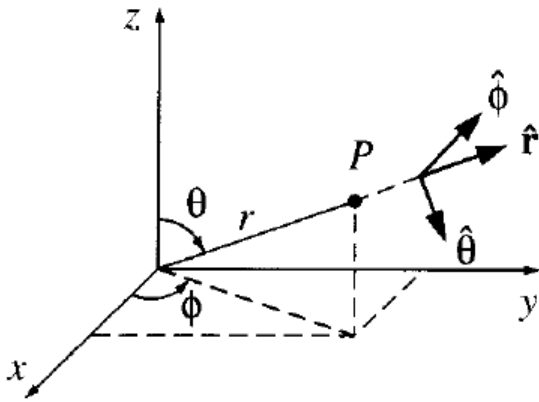# Koordinat Curviliear

Koordinat Bola

Koordinat kutub bola (r, θ, φ) untuk titik P

r = jarak titik P dari pusat koordinat

θ = sudut polar

φ = sudut azimut

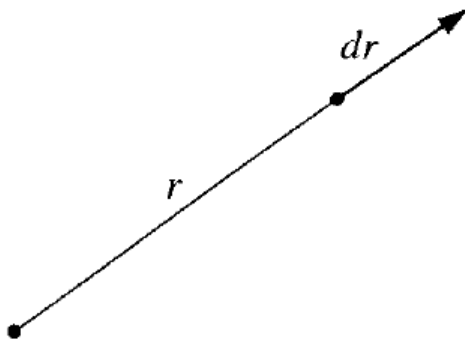Konversi antara koordinat bola (r, θ, φ) dengan koordinat Cartesian (x, y, z) adalah :

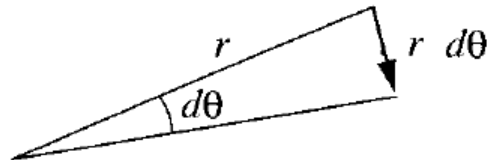$$x = r\sin\theta\cos\varphi$$

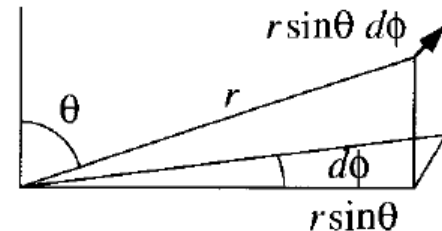$$y = r\sin\theta\sin\varphi$$

$$z = r\cos\theta$$

Vektor Satuan

Tiga vektor satuan dlm koordinat bola adalah **r**, **θ**, φ dengan arah ditunjukkan gambar di bawah ini. Setiap vektor A dlm koordinat bola dpt dinyatakan dengan

$$\mathbf{A} = A_r\,\mathbf{r} + A_\theta\,\boldsymbol{\theta} + A_\varphi\,\varphi$$

(a)   (b)   (c)

Elemen panjang searah r : $dl_r = dr$
Elemen panjang searah θ : $dl_\theta = r\,d\theta$
Elemen panjang searah φ : $dl\varphi = r\sin\theta\,d\varphi$

Secara umum, vektor elemen perpindahan d$\boldsymbol{l}$ adalah :
d$\boldsymbol{l}$ = dr $\boldsymbol{r}$ + r dθ $\boldsymbol{\theta}$ + r sinθ dφ φ

Yang memiliki peran yang sama dengan
d$\boldsymbol{l}$ = dx $\boldsymbol{i}$ + dy $\boldsymbol{j}$ + dz $\boldsymbol{k}$

Elemen Volume

Elemen volume pada koordinat bola adalah perkalian tiga elemen panjang, yaitu
$$d\tau = d\boldsymbol{l_r}\ d\boldsymbol{l_\theta}\ d\boldsymbol{l_\varphi} = r^2 \sin\theta\ dr\ d\theta\ d\varphi$$
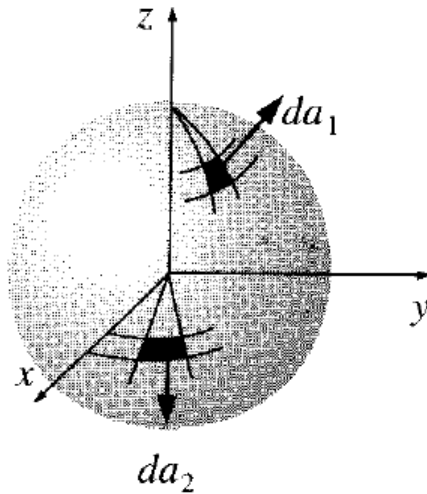
Vektor satuan luas searah r : $da_1 = d\boldsymbol{l_\theta}\ d\boldsymbol{l_\varphi}\ r = r^2\sin\theta\ d\theta\ d\varphi\ \boldsymbol{r}$

Vektor satuan luas searah θ : $da_2 = d\boldsymbol{l_r}\ d\boldsymbol{l_\varphi}\ \boldsymbol{\theta} = r\ dr\ d\varphi\ \boldsymbol{\theta}$

Vektor satuan luas searah φ : $da = d\boldsymbol{l_r}\ d\boldsymbol{l_\theta}\ \varphi = r\ dr\ d\theta\ \varphi$

Gradien, Divergensi, Curl dan Laplacian

Batas integrasi r dari 0 sampai ∞, θ dari 0 sampai π dan φ dari 0 sampai 2π.

Gradien :

$$\nabla T = \frac{\partial T}{\partial r}\,\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\,\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\,\hat{\phi}$$
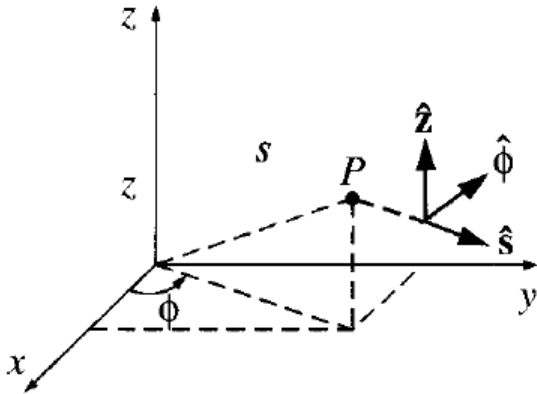
Divergensi :

$$\nabla \cdot v = \frac{2}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta} + \frac{\partial v_\phi}{\partial \phi}$$

Curl:

$$\nabla \times v = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(v_\phi \sin\theta) - \frac{\partial v_\phi}{\partial \phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\phi}$$

Laplacian :

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}$$

Koordinat Silindris

Koordinat silindris (r, φ, z) utk titik P
r = jarak titik P dari pusat koordinat
φ = sudut azimut
z = tinggi titik dasar koordinat

Konversi antara koordinat silindris (r, φ, z) dengan koordinat Cartesian (x, y, z) adalah :

$x = r \cos\varphi$ $y = r \sin\varphi$ $z = z$

Elemen panjang dan volume

Elemen panjang searah r : $dl_r = dr$
Elemen panjang searah ϕ : $dl_\phi = r\, d\phi$
Elemen panjang searah z : $dl_z = dz$

Secara umum, elemen panjang adalah :
$d\boldsymbol{l} = dr\, \boldsymbol{r} + r\, d\phi\, \boldsymbol{\phi} + dz\, \boldsymbol{z}$

Elemen volumen:
$d\tau = d\boldsymbol{l_r}\, d\boldsymbol{l_\phi}\, d\boldsymbol{l_z} = r\, dr\, d\phi\, dz$

Gradien, Divergensi, Curl dan Laplacian

Batas integrasi r dari 0 sampai ∞, φ dari 0 sampai 2π dan z dari - ∞ sampai + ∞.

Gradien :
$$\nabla T = \frac{\partial T}{\partial r}\,\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \phi}\,\hat{\phi} + \frac{\partial T}{\partial z}\,\hat{z}$$

Divergensi :
$$\nabla . v = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_{\phi}}{\partial \phi} + \frac{\partial v_z}{\partial z}$$
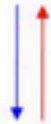
Curl :
$$\nabla \times v = \left[\frac{1}{r}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_{\phi}}{\partial z}\right]\hat{r} + \left[\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rv_{\phi}) - \frac{\partial v_r}{\partial \phi}\right]\hat{z}$$

Laplacian :
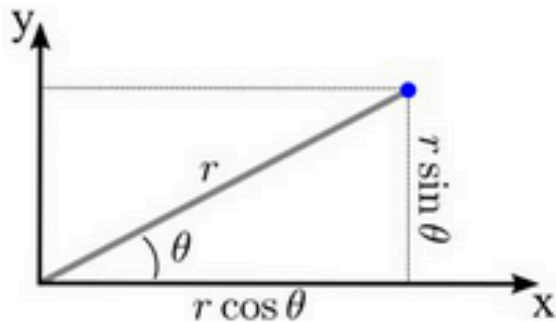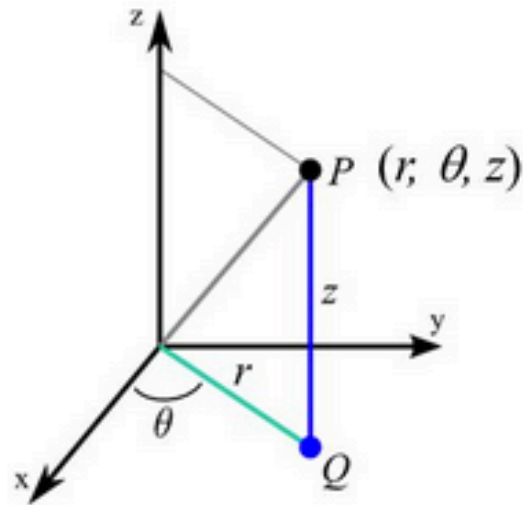$$\nabla^2 T = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Konversi silinder → Kartesian

Koordinat Tabung (Cylindrical)

Koordinat Cartesian

$P\ (r;\ \theta, z)$

z

r

θ

Q

$r\sin\theta$

$r\cos\theta$

θ

r

Tabung ⟶ Cartesian

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

Cartesian → Tabung

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \arctan\left(\frac{y}{x}\right)$$
$$z = z$$

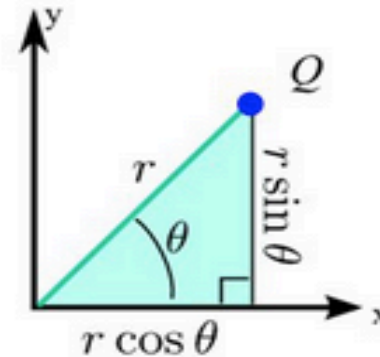# Konversi Bola - Kartesian

Koordinat **Bola (Spherical)** ⟷ Koordinat **Cartesian**

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$
$$\phi = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}}$$
$$\theta = \tan^{-1}\frac{y}{x}$$